The Fabry-Perot Cavity

We will consider for simplicity a symmetric Fabry-Perot cavity. The boundaries of the Fabry-Perot are air (outside, medium 1) glass (inside, medium 2) interfaces. We will use the following notations:

- $\tilde{t}_{12}$ = transmission from outside (1) to inside (2)
- $\tilde{t}_{21}$ = transmission from inside (2) to outside (1)
- $\tilde{r}_{12}$ = reflection from outside (1) to inside (2)
- $\tilde{r}_{21}$ = reflection from inside (2) to outside (1).

The incident field is a plane wave of amplitude unity.

Field transmission

$$\begin{aligned} \mathcal{T} &= \tilde{t}_{12}\tilde{t}_{21}e^{-ikd} \\ &\quad +\tilde{t}_{12}\tilde{t}_{21}\left(e^{-2ikd}\cdot\tilde{r}_{21}\tilde{r}_{21}\right)e^{-ikd} \\ &\quad +\tilde{t}_{12}\tilde{t}_{21}e^{-ikd}\left(e^{-2ikd}\cdot\tilde{r}_{21}\tilde{r}_{21}\right)^2+\ldots \\ &= \tilde{t}_{12}\tilde{t}_{21}e^{-ikd}\frac{1}{1-\tilde{r}_{21}^2e^{-2ikd}}. \end{aligned} \tag{1}$$

Interface properties

For a symmetric interface:

$$\boxed{\tilde{t}_{12}\tilde{t}_{21}-\tilde{r}_{12}\tilde{r}_{21}=1} \tag{2}$$

and

$$\boxed{\tilde{r}_{12}=-\tilde{r}_{21}^*} \tag{3}$$

Equation (2) implies that we can do the following substitution in Eq. (1:

$$\tilde{t}_{12}\tilde{t}_{21}=1+\tilde{r}_{12}\tilde{r}_{21}=1-|r_{12}|^2=1-R. \tag{4}$$

The result for the field transmission is:

$$\boxed{\mathcal{T}(\Omega)=\frac{(1-R)e^{-ikd}}{1-Re^{i\delta}}} \tag{5}$$

where

$$\boxed{\delta(\Omega)=2\varphi_r-2k(\Omega)d} \tag{6}$$

is the total phase shift of a round-trip inside the Fabry-Perot, including the phase shift φ_r upon reflection on each mirror. Note the factor $\exp[-ikd]$ on the numerator, often ignored in most textbooks. It represent a phase shift with no consequence in the case of a pure dielectric. However, kd — hence also δ — is in general a complex number, of which the imaginary part represents absorption or amplification.

Field reflection

$$\begin{aligned}
\mathcal{R} &= \tilde{r}_{12} \\
&\quad +\tilde{t}_{12}\tilde{t}_{21}\tilde{r}_{21}e^{-2ikd} + \tilde{t}_{12}\tilde{t}_{21}\tilde{r}_{21}\tilde{r}_{21}\tilde{r}_{21}e^{-4ikd} + \ldots \\
&= \frac{1}{\tilde{r}_{21}}\left\{\tilde{r}_{12}\tilde{r}_{21} - \tilde{t}_{12}\tilde{t}_{21} + \tilde{t}_{12}\tilde{t}_{21}\left[1 + (\tilde{r}_{21})^2 e^{-2ikd} + \ldots\right]\right\} \\
&= \frac{1}{r_{21}}\left\{-1 + \frac{\tilde{t}_{12}\tilde{t}_{21}}{1 - Re^{i\delta}}\right\} e^{-i\varphi_r}. \qquad (7)
\end{aligned}$$

The phase shift on the numerator is the same as if only the first mirror of the structure was reflecting. Since this phase shift will play no role in the intensity reflection, we will neglect it in the final expression:

$$\boxed{\mathcal{R}(\Omega) = \frac{\sqrt{R}\left(e^{i\delta} - 1\right)}{1 - Re^{i\delta}}.} \qquad (8)$$

One can easily verify that, if — and only if — kd is real:

$$|\mathcal{R}|^2 + |\mathcal{T}|^2 = 1 \qquad (9)$$

Equations (5) and (8) are the transfer functions for the Fourier transform of the field. The dependence on the frequency argument Ω occurs through $k = n(\Omega)\Omega/c$.

Unequal reflectivities

Reflection

We use the same notations as above, but label the parameters of the second surface with a “prime”.

$$\begin{aligned}
\mathcal{R} &= \tilde{r}_{12} \\
&\quad +\tilde{t}_{12}\tilde{t}_{21}\tilde{r}'_{21}e^{-2ikd} + \tilde{t}_{12}\tilde{t}_{21}\tilde{r}'_{21}\tilde{r}_{12}\tilde{r}'_{21}e^{-4ikd} + \ldots \\
&= \frac{1}{\tilde{r}_{21}}\left\{\tilde{r}_{12}\tilde{r}_{21} - \tilde{t}_{12}\tilde{t}_{21} + \tilde{t}_{12}\tilde{t}_{21}\left[1 + (\tilde{r}_{21}\tilde{r}'_{21})e^{-2ikd} + (\tilde{r}_{21}\tilde{r}'_{21})^2 e^{-4ikd}\ldots\right]\right\} \\
&= \frac{1}{r_{21}}\left\{-1 + \frac{\tilde{t}_{12}\tilde{t}_{21}}{1 - (\tilde{r}_{21}\tilde{r}'_{21})e^{-2ikd}}\right\} e^{-i\varphi_r}. \qquad (10)
\end{aligned}$$

Substituting $\tilde{t}_{12}\tilde{t}_{21} = 1 + \tilde{r}_{12}\tilde{r}_{21}$:

$$\begin{aligned}\mathcal{R} &= \frac{1}{r_{21}}\left\{\frac{(\tilde{r}_{21}\tilde{r}'_{21})e^{-2ikd} - 1 + 1 + \tilde{r}_{12}\tilde{r}_{21}}{1-(\tilde{r}_{21}\tilde{r}'_{21})e^{-2ikd}}\right\}e^{-i\varphi_r} \\ &= \frac{(\tilde{r}'_{21})e^{-2ikd} + \tilde{r}_{12}}{1-(\tilde{r}_{21}\tilde{r}'_{21})e^{-2ikd}} \end{aligned} \tag{11}$$

Defining:

$$\begin{aligned} \tilde{r}'_{21} &= r'e^{i\varphi_{r'}} \\ \tilde{r}_{21} &= re^{i\varphi_r} \\ \tilde{r}_{12} = -\tilde{r}^*_{21} &= -re^{-i\varphi_r} \\ \delta &= -2kd + \varphi_{r'} + \varphi_r + i\alpha \end{aligned} \tag{12}$$

were α is a loss coefficient (dimensionless). we get the final expression for the reflection of unbalanced Fabry-Perot in reflection:

$$\boxed{\tilde{\mathcal{R}}(\Omega) = \frac{(r'e^{i\delta} - r)e^{-i\varphi_r}}{1 - rr'e^{i\delta}}.} \tag{13}$$

1 Gires-Tournois and ring resonator

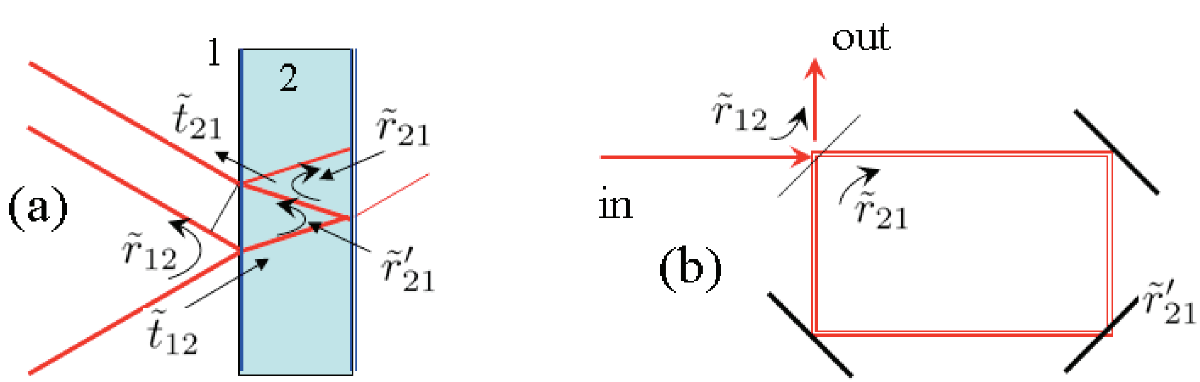

Figure 1: (a) Fabry Perot with field reflectivity r from the left, and r' from the right. (b) This configuration is equivalent to a ring resonator.

The Fabry-Perot with a 100% reflectivity at r' is a Gires-Tournois resonator, which is equivalent to a ring resonator, as shown in Fig. 1, The intensity reflection corresponding to Eq. 13 is:

$$R = |\mathcal{R}|^2 = \frac{r'^2 + r^2 - 2rr'\cos\delta}{1 + r^2r'^2 - 2rr'\cos\delta} \tag{14}$$

It can be easily verified that this expression is equal to unity when $R' = 1$ and $\alpha = 0$. It is worth noting also that the coefficient r' place exactly the same role as an absorption loss α in Eq. 14. Therefore, the ring resonator can be brought to $r' \approx 1$ by inserting gain in the ring.

It is interesting to assess the change in reflectivity R at resonance, when the reflectivity r' is less than unity $r' = 1 - \epsilon$. To first order in ϵ, one finds $R \approx 1 - 2\epsilon(1-r^2)/(1-r)^2$. While this approximation is only valid for small ϵ, it does show that the higher the quality factor of the resonator, the larger the departure from unit of the output at resonance.

1.1 Dispersion

The transfer function of the Fabry-Perot or ring resonator given by Eq. (13) is $\sqrt{R}\exp(-\psi)$, where R is given by Eq. (14). T find ψ we decompose $\tilde{\mathcal{R}}(\Omega)$ in its real and imaginary part:

$$Re(\mathcal{R}) = \frac{r'(1+r^2)\cos\delta - r(1+r'^2)}{1 + r^2 r'^2 - 2rr'\cos\delta} \tag{15}$$

$$Imag(\mathcal{R}) = \frac{(r' - r^2 r')\sin\delta)}{1 + r^2 r'^2 - 2rr'\cos\delta} \tag{16}$$

The phase angle of the transfer function is:

$$\psi = \varphi_r - \arctan\left\{\frac{r'(1-r^2)\sin\delta}{r'(1+r^2)\cos\delta - r(1+r'^2)}\right\} = \varphi_r - \arctan\left\{\frac{a\sin\delta}{b\cos\delta - c}\right\} \tag{17}$$

The dispersion of the device is the first derivative with respect to frequency:

$$\frac{d\psi}{d\delta} = -\frac{a(b - c\cos\delta)}{a^2\sin^2\delta + (b\cos\delta - c)^2}. \tag{18}$$

where

$$a = r'(1-r^2) \tag{19}$$

$$b = r'(1+r^2) \tag{20}$$

$$c = r(1+r'^2) \tag{21}$$

For the no-loss case $r' = 1$:

$$\frac{d\psi}{d\delta} = \frac{(1-r^2)(1+r^2-2r\cos\delta)}{(1-r^2)^2\sin^2\delta + [(1+r^2)\cos\delta - 2r]^2}. \tag{22}$$

This expression at resonance($\delta = 0$)leads to the well known dispersion of the Gires-Tournois:

$$\frac{d\psi}{d\Omega} = \frac{1+r}{1-r}\frac{d\delta}{d\Omega}. \tag{23}$$

1.2 Plots and trends

The four plots of Fig. 2 illustrate some of the dependence of the dispersion and resonant reflection on r and r'. The dependence in reflectivity of the front face or on the coupling constant r, for r' constant is that increasing r results in:

- Larger loss.
- Skinnier resonance.
- Larger beat note enhancement.

As for the dependence in r': setting $r' < 0.9$ results in the reflection going to zero at resonance. Increasing r' results in

- Decreasing loss at resonance.
- Slightly broader resonance.
- Slightly lower enhancement.

r and r' are *field* reflectivities. The value of $r' = 0.999$ corresponds thus to an intensity reflectivity of 99.8%, which is a relatively standard coating. A scenario with $r = 0.96$ and $r = 0.999$ gives a large enhancement (50) with a loss of less than 10%.

It is rather strange for me that r' contains gain or loss in the case of the non-symmetric Fabry-Perot, but once $r' = r$, that is no longer the case (see below).

Another surprising observation is that the dispersion has opposite sign for $r' > r$ as compared to $r' < r$.

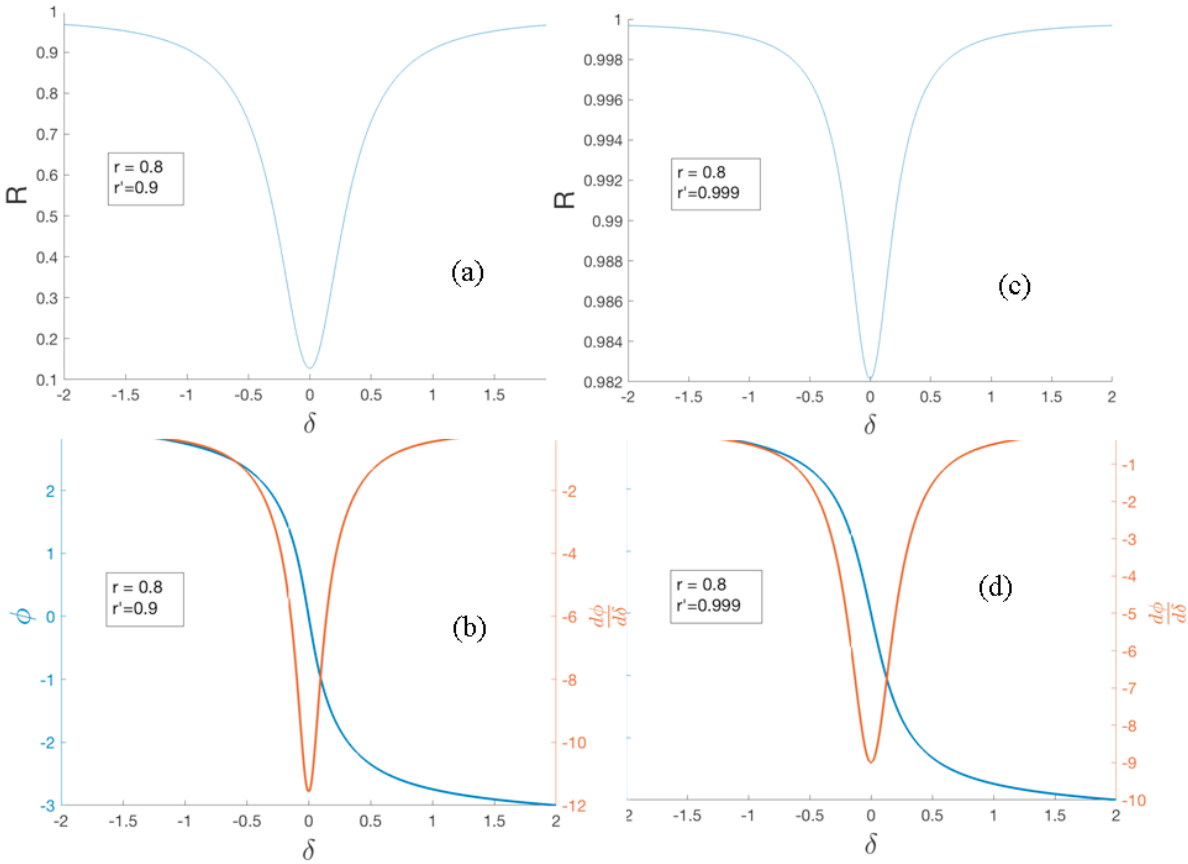

Figure 2: (a) Intensity reflection versus round-trip phase δ, for field reflectivities $r = 0.8$ and $r' = 0.9$. We note that the reflection drops nearly to zero at resonance. (b) Phase (blue line) and dispersion (red line = derivative of the phase) versus δ. (c)Intensity reflection for $r = 0.8$ and $r' = 0.999$. (c) phase and dispersion for $r = 0.8$ and $r' = 0.999$.

Fabry-Perot with gain

We assume the gain to be linear and uniform in the frequency range around a Fabry-Perot resonance of interest. The gain factor a at each round-trip can be incorporated into the phase factor δ: $\tilde{\delta} = \delta - ia$, where $\tilde{\delta}$ is the previously defined complex phase factor, and δ is the real part of this phase factor.

Substituting into the expressions for the transmission [Eq. (5)] and reflection [Eq. (8)] and taking the absolute value squared to find the intensity transmission factor:

$$
\begin{aligned}
|\mathcal{R}|^2 &= \frac{(1-R)^2}{1 + Re^{2a} - 2Re^{a}\cos\delta} \\
|\mathcal{T}|^2 &= \frac{R\left[e^{2a} + 1 - 2e^{a}\cos\delta\right]}{1 + Re^{2a} - 2Re^{a}\cos\delta}.
\end{aligned}
\tag{24}
$$

The traditional approach to simplify these expression is to make the substitution $\cos\delta =$

$1 - 2\sin^2(\delta/2)$. The result is:

$$\boxed{\begin{aligned} |\mathcal{R}|^2 &= \frac{R\left(e^a-1\right)^2}{\left(1-Re^a\right)^2}\frac{\left[1+\frac{4e^a}{(e^a-1)^2}\sin^2\frac{\delta}{2}\right]}{1+\frac{4Re^a}{(1-Re^a)^2}\sin^2\frac{\delta}{2}} && (25) \\ |\mathcal{T}|^2 &= \frac{\left(1-R\right)^2}{\left(1-Re^a\right)^2}\frac{1}{1+\frac{4Re^a}{(1-Re^a)^2}\sin^2\frac{\delta}{2}} && (26) \end{aligned}}$$

For values of δ close to the resonance condition ($N\pi$), the shape of the transmission function is a Lorentzian, of half width determined by the condition:

$$\frac{4Re^a}{(1-Re^a)^2}\sin^2\frac{\delta}{2} = 1. \tag{27}$$

As the gain a increases, the transmission at resonance increases, to reach infinity at threshold ($Re^a = 1$). The linewidth decreases to zero at the threshold gain.

The reflection at $\delta = 0$ is:

$$|\mathcal{R}|^2 = \frac{R\left(1-e^a\right)^2}{(1-Re^a)^2} \tag{28}$$

For small a, this expression can be approximated as:

$$\frac{a^2R}{(1-Re^a)^2} \tag{29}$$

which also tends to infinity at threshold.

Let us sent through this Fabry-Perot a probe beam, at a frequency tuned to the half width (i.e. transmission of the intensity = 0.5) of the empty (no gain) Fabry-Perot. The detuning for the empty cavity is given by the condition:

$$\sin^2\frac{\delta}{2} = \frac{(1-R)^2}{4R}. \tag{30}$$

If this value is substituted in the expression for the transmission, we find:

$$|\mathcal{T}|^2 = x\left[\frac{1}{1+xe^a}\right] \tag{31}$$

The transmission is the limit for $x \to \infty$ of this expression, where

$$x = \frac{(1-R)^2}{(1-Re^a)^2}. \tag{32}$$

The solution is

$$\frac{x}{1+xe^a} \to e^{-a} \approx 1. \tag{33}$$

Thus, even though the linewidth tends to zero and the transmission to infinity, there is still a transmission factor of unity at the optical frequency corresponding to 50% transmission for the empty cavity.

The intensity inside the resonator is simply the transmitted intensity divided by the transmission coefficient of the second mirror:

$$I_i = \frac{I_0}{|\mathcal{T}|^2} \frac{1}{1-R}. \tag{34}$$

The threshold condition $Re^a = 1$ corresponds to the onset of laser oscillation. In the considerations above, we have assumed the gain a to be a constant, independent of the intensity. In reality, the gain a is itself a function of I_i:

$$a = \frac{a_0}{1 + \frac{I_i}{I_s}}, \tag{35}$$

where a_0 is the unsaturated absorption coefficient, and I_s is the saturation intensity.

Given an unsaturated gain above threshold, and an input intensity I_0, the intensity will build up inside the cavity, and the gain will decrease, until reaching the equilibrium condition that corresponds to the gain = loss:

$$Re^{a_0/(1+I_i/I_s)} = 1 \tag{36}$$

which is the threshold condition defined previously through Eqs. (25) and (26).

Absorbing Fabry-Perot

The absorbing Fabry-Perot equivalent to the Fabry-Perot with gain, except that the gain factor a is negative.

Impulse response of a Fabry-Perot

The transmission of any signal — cw, pulse, train of pulses — through a Fabry-Perot can be calculated by taking the product of the Fourier transform of the pulse and the transfer function Eq. (5). The result of this operation is the Fourier transform of the transmitted pulse. For instance, cw radiation corresponds in the frequency domain to a $\delta(\Omega-\omega_\ell)$ function. The transmitted field through the Fabry-Perot is therefore simply the value of the function (5) at the frequency ω_ℓ ($\times\mathcal{E}_0$). For a very short pulse that covers N modes of the Fabry-Perot, the Fourier transform of the transmission is a comb of N Fabry-Perot transmission peaks. The Fourier transform of that frequency comb is a pulse train. Because the envelope of these peaks has the same shape as the Fourier transform of the incident pulse, the pulses of the sequence have the same shape and duration as the incident pulse. The shape of the envelope of the pulse sequence is the inverse Fourier transform of a Fabry-Perot transmission peak. Since the latter is a Lorentzian, the pulse train follow a decaying exponential. The latter situation is easier to analyze in the time domain. A single pulse, shorter than the mirror spacing, is sent to the Fabry-Perot. There are no interferences. The pulse rattles inside the cavity, losing a fraction $(1-R)$ at each reflection.

We consider next the more complex problem of the transmission/reflection of a Fabry-Perot to a train of pulses. Here again, the numerical approach would be to multiply the Fourier spectrum of the pulse train by the transfer function Eq. (5) for transmission, or Eq. (8) for the reflection. The direct derivation in time however gives a better understanding of the resonator response.

We consider the irradiation of a Fabry-Perot by the output of a mode-locked laser of which the cavity is 4 $\times$ longer than the Fabry-Perot. Each pulse of the train has a duration much smaller than the round-trip time of the Fabry-Perot.

Let us first derive an expression for the electric field of the transmitted pulse(s) through this Fabry-Perot irradiated by the mode-locked laser. We will make step by step derivation, following the pulse as it enters the resonator and travels back and forth in that cavity, keeping in mind the ratio of cavity and pulse rates.

The field transmission factor is given by:

$$\begin{aligned}\mathcal{T} &= \tilde{t}_{12}e^{-ikd}\tilde{t}_{21}+\tilde{t}_{12}\tilde{r}_{21}^{8}e^{-9ikd}\tilde{t}_{21}+\tilde{t}_{12}\tilde{r}_{21}^{16}e^{-19ikd}\tilde{t}_{21}\dots \\ &= \frac{\tilde{t}_{12}\tilde{t}_{21}e^{-ikd}}{1-\tilde{r}_{21}^{8}e^{-i8\delta}} \qquad (37)\end{aligned}$$

For the intensity transmission:

$$|\mathcal{T}|^2=\frac{(1-R)^2}{1+R^8-2R^4\cos 8\delta}. \qquad (38)$$

If we make the standard replacement $\cos 8\delta = 1-2\sin^2 4\delta$, the equation for the transmitted intensity becomes:

$$|\mathcal{T}|^2 = \frac{(1-R)^2}{1+R^8-2R^4(1-2\sin^2 4\delta}$$

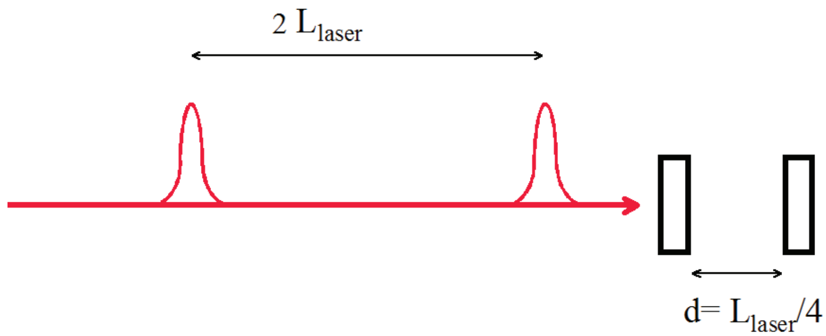

Figure 3: Fabry-Perot cavity irradiated by a mode-locked pulse train of 4 times lower repetition rate than the round-trip rate of that Fabry-Perot.

$$
\begin{aligned}
&= \frac{1}{(1+R+R^2+R^3)^2 + \frac{4R^4}{(1-R)^2}\sin^2 4\delta} \\
&= \frac{1}{(1+R+R^2+R^3)^2}\frac{1}{1+\frac{4R^4}{(1-R)^2(1+R+R^2+R^3)^2}\sin^2 4\delta}
\end{aligned}
\tag{39}
$$

This is to be compared to the transmission of the Fabry-Perot for cw light:

$$
\left[|\mathcal{T}|^2\right]_{\rm cw} = \frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}. \tag{40}
$$

In the case of the cw FP, this represents near the resonance (for R close to unity) a Lorentzian of full width at half maximum (FWHM):

$$
\Delta\delta_{cw} = \frac{1-R}{\sqrt{R}} = \frac{T}{\sqrt{R}}. \tag{41}
$$

For this case of mode-locked input, the FWHM of the Lorentzian is:

$$
\Delta\delta_{ml} = \frac{(1-R)(1+R+R^2+R^3)}{8R^2} \approx \frac{T}{2R^2}. \tag{42}
$$

where the approximation $R \approx 1$ was made for the sum in the numerator. One could argue that the Fabry-Perot is 4 times longer as seen by the mode-locked train, therefore the

FWHM should be 4 times narrower. On the other hand, the losses due to the reflections not synchronized with the feeding pulse contribute to a broadening of the transmission peak (term in R^2 instead of $\sqrt{R}$ in Eq. (42).

The Fabry-Perot transmission at resonance is the expression Eq. (38) for $\delta = 0$:

$$\begin{aligned} |\mathcal{T}|^2 &= \frac{(1-R)^2}{(1-R^4)^2} \\ &= \left(\frac{1}{1+R+R^2+R^3}\right)^2 \approx \frac{1}{16}, \end{aligned} \tag{43}$$

where the approximation applies to a high Q cavity ($R \approx 1$).

Reflected and transmitted energy in the limit $R \to 1$

Let W_1 be the energy of each incident pulse. Since there are four transmitted pulses for each incident pulse, the total transmitted energy is $4W_1/16 = W_1/4$, where W_1 is the energy of incident light. The reflected energy should therefore be $(3W_1/4)$, which comprises 3 pulses with energy $W/16$ and one pulse with energy $W_1 \times [(12/16) - (3/16)] = 9W_1/16$, in order to satisfy energy conservation. The reflected field of $3\mathcal{E}_1/4$ results from destructive interference of the incident field (amplitude $\mathcal{E}_1$) and the transmitted field from the Fabry-Perot (amplitude $\mathcal{E}_1/4$).

The nonlinear Fabry-Perot

Saturable gain of absorption off resonance

As we have seen, the laser can be seen as a nonlinear Fabry-Perot, where the linewidth and tranmission factor are dependent on the gain, itself dependent on the intracavity intensity, itself dependent on the transmission factor. If radiation is injected in such a Fabry-Perot with gain off resonance, the index of refraction is also dependent on the intensity, hence on the transmission, which is also dependent through δ on the index... One can feel that this problem can become rather complex.

Nonlinear index of refraction

We will consider here a simpler nonlinearity, that of the Kerr effect, or an index of refraction linearly dependent on the intensity:

$$n = n_0 + n_2 I_i \tag{44}$$

where the subscript i of the intensity indicates that we are concerned with the intracavity intensity.

Fabry-Perot – simplest case

We will consider that the phase shift δ is real, and is only due to the propagation factor: $\delta = -kd - \phi$. The intensity transmission of a Fabry-Perot is given by:

$$\begin{aligned} T &= \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\phi}{2}} \\ &= \frac{1}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2\frac{\phi}{2}} \\ &\approx \frac{1}{1 + \frac{R}{(1-R)^2}\phi^2}. \end{aligned} \tag{45}$$

where the approximation is valid for $(1-R) \ll 1$, and $\phi \ll 1$. Within that approximation, the transmission function is a Lorentzian of width (FWHM) $\Delta\phi_t$:

$$\Delta\phi_t = 2\frac{\sqrt{R}}{(1-R)}. \tag{46}$$

In the above expressions, $R = \sqrt{R_1 R_2}$, R_1 and R_2 being the (intensity) reflection coefficients of the mirrors, and $\phi = k(2L) = 4\pi nL/\lambda = 4\pi\nu nL/c$ is the round-trip phase shift. The free spectral range, or the frequency spacing between transmission peaks, is $\Delta\nu = c/(2nL)$. The finesse F is the ratio of the full width half maximum of each transmission peak $\Delta\nu_t$ to the free spectral range $\Delta\nu_{ax} = c/(2nL)$.

$$F = \frac{\pi\sqrt{R}}{1-R} \tag{47}$$

Another form for the Fabry-Perot transmission is:

$$T = \frac{(1-R)^2}{1+R^2-2R\cos\phi} = \left[\frac{\epsilon}{\mathcal{D}}\right]^2. \tag{48}$$

The transmission of the Fabry-Perot can be determined by a simple geometrical construction, as sketched in Fig. 4(b). The incident field is represented by a unit vector OA with a phase angle ϕ. The quantity $\mathcal{D} = \sqrt{1-2R\cos\phi+R^2}$ is the side AB of the triangle, where B is the point at $\epsilon = 1-R$ from the intersection of the circle with the $\phi = 0$ axis. It is easily seen from the figure that the transmission factor $[\epsilon/\mathcal{D}]^2$ varies between unity and $[(1-R)/(1+R)]^2$.

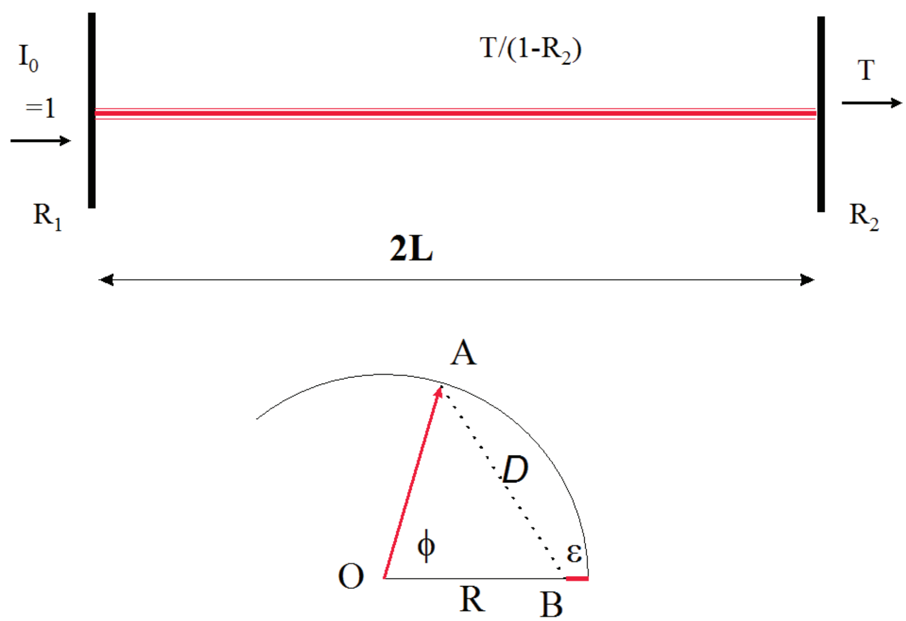

Figure 4: Fabry-Perot cavity (a) and geometrical representation of its transmission factor (b).

A Fabry-Perot cavity as an ultrafast switch?

Any change in cavity length, or change in index, will shift the resonance. Since the resonance can be very narrow, one would see here an ideal opportunity to make an ultrafast switch. Let us assume for simplicity that the two mirrors are identical. For the transmitted intensity T given above, the intensity inside the device is $T/(1-R)$. If the Fabry-Perot is filled with a medium with a nonlinear index ($n = n_0 + n_2 I$), the phase shift ϕ is a function of the transmission:

$$\phi = \phi_0 + \frac{4\pi n_2 T I_0 L}{\lambda(1-R)} = \phi_0 + \frac{T I_0}{a}. \tag{49}$$

Equation (49) can be represented by straight lines on a graph of T versus ϕ:

$$T = \frac{a}{I_0}\left(\phi - \phi_0\right). \tag{50}$$

Starting from the situation sketched in Fig. 5, as the incident intensity increases from an initial value of 0 [vertical line for Eq. (50)] the transmission become multivalued. The Fabry-Perot has become a multistable element. The intersections of the straight lines in Fig. 5 with the Fabry-Perot transmission curves are made to construct the transmission characteristic of the bistable Fabry-Perot of Fig. 6.

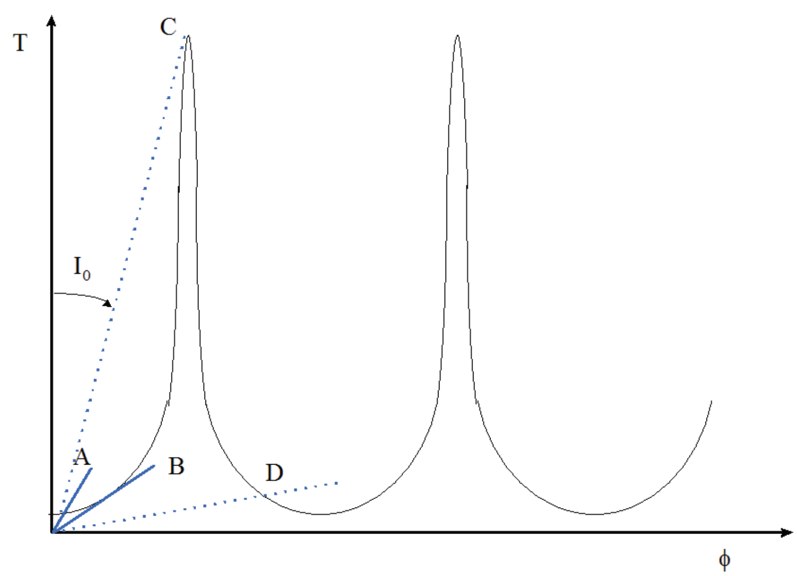

Figure 5: Fabry-Perot transmission versus the phase factor ϕ (a), which has become itself a linear function of transmission (straight lines of decreasing slope as the intensity I_0 is increasing).

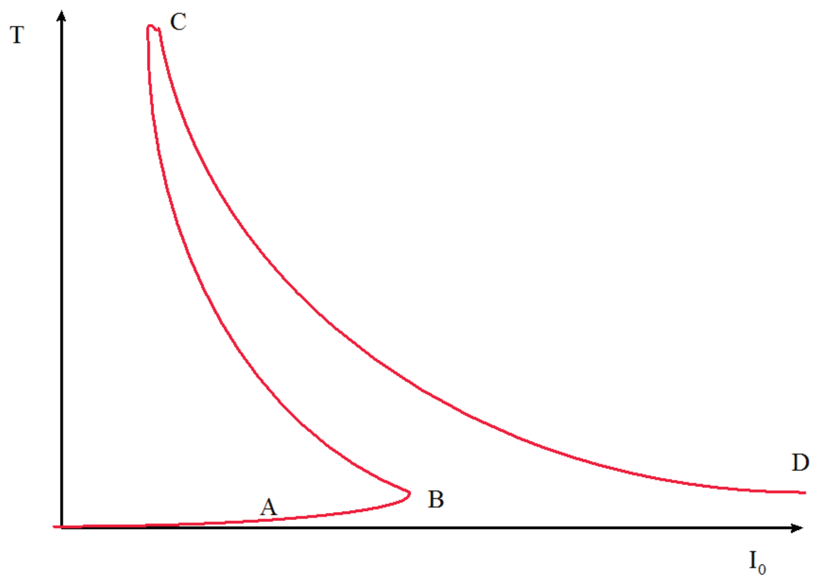

Figure 6: Fabry-Perot transmission versus the input intensity, constructed graphically from the previous figure. Corresponding points A, B, C and D are labelled on both figures.