Grating selection

You are asked to select a square flat grating 10 cm wide, for best wavelength selection (in first order) at 600 nm. The clearance is such that the acceptance angle for the incident beam ranges from -89^o to +89^o.

1. What is the configuration (angle of incidence and diffraction) and groove density that would give you the best resolution?

2. Assuming you can resolve a deviation angle of 1^o, what is the corresponding resolution in wavelength $\Delta\lambda_1$?

3. What is the diffracted angle (first order) at the second harmonic (300 nm) and the resolution $\Delta\lambda_2$ corresponding to a deviation angle of 1^o?

4. Can you achieve the same resolution at 300 nm (rather than for 600 nm) in higher order (if so, which order)?

Solution

1. The resolution is typically connected to the number of grooves that are illuminated. The smaller the groove, the larger the number of grooves over the dimension of the grating. In the grating equation:
$$\sin\alpha + \sin\beta = \frac{\lambda}{d}. \tag{1}$$
For a given λ, the smallest groove will correspond to the largest left hand side. The maximum value is $\sin\alpha = \sin\beta \approx 1$, i.e. the closest possible to grazing incidence (which is here 89°). The optimum configuration is thus $d = \lambda/2$ in first order. This (extreme) Littrow configuration has been used for making very narrow bandwidth dye laser, and narrow line external cavity semiconductor lasers.

2. Taking the derivative of Eq. (1):
$$\cos\beta\Delta\beta = \frac{\Delta\lambda_1}{d} = 2\frac{\Delta\lambda_1}{\lambda}, \tag{2}$$
which leads to the resolution:
$$\Delta\lambda_1 = \frac{1}{2}\lambda \times \cos\beta \times \Delta\beta = 0.00015\lambda = 0.09\text{nm}$$

3. At the second harmonic, $\lambda = d$, hence $\sin\beta \approx 0$. Using Eq. 2) again:
$$\cos\beta\Delta\beta \approx \Delta\beta = \frac{\Delta\lambda_2}{d} = \frac{\Delta\lambda_1}{\lambda}, \tag{3}$$
which leaves us with the resolution:
$$\Delta\lambda_2 = \lambda \times \Delta\beta = 0.017\lambda \approx 5\text{nm}$$

4. The grating equation (1) in second order at 300 nm is identical to the one at 600 nm in first order. Therefore we get the same resolution ins second order at 300 nm than in first order at 600 nm (Littrow and grazing incidence).