Nonlinear Optics 2024 — Homework 4

Due Wednesday, March 20, 2024

Gaussian beams with phase conjugated mirror

Reminder: equation of a Gaussian beam:

$$\tilde{\mathcal{E}}(x,y,z) = \frac{\mathcal{E}_0}{\sqrt{1+z^2/\rho_0^2}} e^{-i\Theta} \times e^{-ik_\ell (r^2)/2R} \times e^{-(r^2)/w^2} \tag{1}$$

where the various parameters are:

$$\begin{aligned} r^2 &= x^2+y^2 \\ R = R(z) &= z + \rho_0^2/z \\ w = w(z) &= w_0\sqrt{1+z^2/\rho_0^2} \\ \Theta = \Theta(z) &= \arctan(z/\rho_0) \\ \rho_0 &= \rho(z=0) = \frac{n\pi w_0^2}{\lambda}. \\ \rho &= \frac{n\pi w^2}{\lambda} \end{aligned}$$

A flat, ideal phase conjugate mirror transforms Eq. (1) into:

$$\tilde{\mathcal{E}}(x,y,z) = \frac{\mathcal{E}_0}{\sqrt{1+z^2/\rho_0^2}} e^{i\Theta} \times e^{ik_\ell (r^2)/2R} \times e^{-(r^2)/w^2} \tag{2}$$

In terms of complex q-parameter, Eq. (1) is:

$$\tilde{\mathcal{E}}(x,y,z) = \frac{\mathcal{E}_0}{\sqrt{1+z^2/\rho_0^2}} e^{-i\Theta} \times e^{-ik_\ell (r^2)/2\tilde{q}} \tag{3}$$

1.Find the q-parameter transformation for a flat phase conjugated mirror

2. Write this transformation in terms of ABCD matrix

Without phase conjugation:

$$\frac{1}{q_2} = \frac{C + \frac{D}{q_1}}{A + \frac{B}{q_1}} \tag{4}$$

Hint: with phase conjugation the expression may involve complex conjugation.

3. Stability condition of a simple cavity

Having determined the ABCD matrix for a phase congution mirror, apply this result to analyze the satbility of a simple cavity with a curved (normal) mirror and a flat phase conjugated mirror.
Hint: make 2 round-trips.
Find the stability condition and the beam size.

4. Why the hint in the previous question?

5. Non degenerate FWM: find the resonance condition for longitudinal modes

The process is:

$$\omega_p + \omega_p - \omega_1 - \omega_2 = 0. \tag{5}$$