1 Optics 463 — Homework for Wednesday, September 2, 2020

1.1 From second order Maxwell to first order with nonlinear index

1.1.1 Part 1

The purpose is to make appropriate slowly varying approximations on the second order Maxwell propagation equation in cylindrical coordinates, to get to the first order wave equation. The starting (second order) propagation equation is:

$$\left[\Delta_{tr} + \partial^2_{zz} - \frac{n^2}{c^2}\partial^2_{tt}\right]\mathcal{E}(r,z)e^{(i\omega t - kz)} = 0. \tag{1}$$

An important simplification: $\mathcal{E}(r,z)$ is NOT a function of time. Δ_{tr} is the transverse Laplacian, which, in cylindrical coordinates (cylindrical symmetry) is:

$$\Delta_{tr} = \partial^2_{rr} + \frac{1}{r}\partial_r. \tag{2}$$

Additional twist: a nonlinear (intensity dependent) index of refraction:

$$n = n_0 + n_2 I = n_0 + n_2\frac{\mathcal{E}^2}{2\eta}, \tag{3}$$

where η is the characteristic impedance of the medium. You will make the approximation that n_2 is small (not zero!)

1.1.2 Part 2: Normalization

Re-write the final equation of part 1 in dimensionless units, using for the field $F = \mathcal{E}/\mathcal{E}_0$ where $\mathcal{E}_0 = \sqrt{\frac{\eta}{n_2}}$, and for the distance $\chi = kr$.

1.2 Spacecraft

In the 70's at the peak of excitement about space exploration, a propulsion scheme was proposed to visit another galaxy. Since it is extremely costly to bring up fuel to a spacecraft, it was proposed to attach huge reflectors to the spacecraft, and to "push" it with a powerful light beam. Another approach is to "beam up" energy with a powerful laser, which is then collected by solar panels, and converted into energy to power an ionic engine to propel the satellite or spacecraft.

1. To get a comparison between the two approaches, assume that the energy conversion is 10% efficient (from laser light to propulsion). Assume a continuous laser beam of 1 MW power, being completely collected by solar panels of 100 m diameter, and applied for 200 seconds. Assuming a mass of 1000 kg, what will be the velocity increase of the spacecraft after 200 s or irradiation?

2. For comparison, replace the solar panels by perfectly reflecting mirrors. The laser beam is now used to push the spacecraft by radiation pressure. What will be the velocity after 200 seconds (still a 1 MW laser beam)?

Note that the authors of the article had also an answer to the question: "how do you stop the spacecraft". Simple, answered the authors: the space travelers have to attempt communication with a more advanced civilization, screaming "help" in all possible language and form as they are pushed into space!