

Maxwell's equations

In dielectrics (no charge, no current):

$$\begin{aligned}\nabla \cdot D &= \rho = 0 \\ \nabla \cdot B &= 0 \\ \nabla \times E + \frac{\partial B}{\partial t} &= 0 \\ \nabla \times H - \frac{\partial D}{\partial t} &= J = 0\end{aligned}\tag{1}$$

Medium equation:

$$D = \epsilon_0 E + P_L + P_{NL} = \epsilon_0(1 + \chi E) + P_{NL} = \epsilon E + P_{NL}.$$

$$\begin{aligned}\nabla \times \left\{ \nabla \times E + \frac{\partial B}{\partial t} \right\} &= 0 \\ \nabla \times (\nabla \times E) + \nabla \times \frac{\partial B}{\partial t} &= 0 \\ \nabla(\nabla \cdot E) - \nabla^2 E + \nabla \times \frac{\partial B}{\partial t} &= 0\end{aligned}\tag{2}$$

$$\nabla \times \frac{\partial B}{\partial t} = \mu_0 \frac{\partial^2 D}{\partial t^2} = \mu_0 \epsilon \frac{\partial^2 E}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

In a *linear* medium:

$$\nabla^2 E - \mu_0 \epsilon \frac{\partial^2 E}{\partial t^2} = 0.$$

In terms of the index of refraction:

$$\nabla^2 E - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$