## Micromechanical forces

The mechanical action of a laser beam is not limited to pushing, it can also twist. Circularly polarized light corresponds to an electric field spinning around the light ray, at a rate of the light frequency. Right-circular polarization will be spinning to the right, left-circular spinning to the left. The photon can be seen as an elementary bullet spinning about the axis of propagation. Rifles use spin-stabilized bullets: The barrel's rifling imparts spin to the bullet as it passes through the bore, a rotation meant to stabilize the bullet in flight. Indeed, we know that angular momentum is conserved, so the spinning motion of the bullet will tend to maintain its axis pointing towards the direction of propagation. As the bullet hits a body, its momenta are exchanged with the target: the linear momentum producing a recoil (the equivalent of radiation pressure $h\nu/c$ for the photon) and the angular momentum producing a torque. The same phenomena of momenta exchanges occur at the source: the gun experiences a recoil when fired, and a torque in the opposite direction as that of the spin. The angular momentum $L$ and the kinetic energy $K$ of a object of mass $m$ spinning at a distance $r$ from an axis, at a rate of $f$ turns/second[1], are respectively $L = m(2\pi f)r^2 = m\Omega r^2$ and $K = m(2\pi f)^2 r^2 = m\Omega^2 r^2$. The photon is the ultimate elementary bullet: despite the fact that it has no mass, it has a linear momentum ($mv$ for the bullet) which is $h\nu/c$, and, as any spinning bullet, an angular momentum $h/2\pi$. One may wonder if the angular momentum of a circularly polarized laser pointer may give some strange sensation to its user. Would it act as a gyroscope, moving the beam sideways when one wants to move the spot downwards? Or would one feel a recoil torque on the laser when turning on the laser? To get a sense of the magnitude of the angular momentum of light, let us imagine a green laser pointer emitting circularly polarized light, as in Fig. 1 (a).To the angular momentum $Nh/(2\pi)$ (where $N$ is the number of photons) corresponds the kinetic energy of:

$$\frac{dN}{dt}\frac{h}{2\pi} = \frac{d(Nh\nu)}{dt}\frac{1}{2\pi\nu} = \frac{WT}{2\pi}, \tag{1}$$

where $W$ is the laser power, and $T \approx 2$ fs the period of the laser light oscillation. Even with a laser pointer of 20 kW power (!!), we find that the kinetic energy associated with all the photons spinning in the same direction is only $\ldots 5 \cdot 10^{-12}$ J. The mechanical system with the same kinetic energy would be a mass of 1 gram, held at arms length (1 m) by a man (or polar bear) standing on the North pole [hence a rotation rate of $\Omega = (2\pi$ radian$)/$day $= 0.00007$ radian/s]. In microresonators where similar powers are circulating, that kinetic rotational energy is concentrated in a volume of the order of a $\mu$m$^3$, thus an energy density of 5 J/cm$^3$, quite sizable for a micromechanical structure.

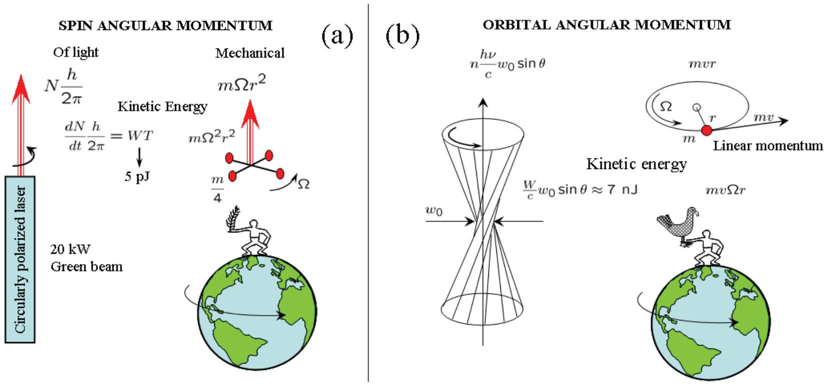

Figure 1: (a) What is the angular momentum associated with a circularly polarized green laser pointer of 20 kW power? The angular momentum of the beam is the value per photon $[h/(2\pi)]$ times the number of photon $N$. The kinetic energy, rate of change of angular momentum, is equal to the product of the laser power $W$ by the light period $T \approx 1.5$ fs divided by $2\pi$. This kinetic energy is that of a 1 g feather on a 1 m arm, spinning at the rate of 1 turn/day! (b)

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[1] $\Omega = 2\pi f$ is the angular frequency, in radians/second.

It is possible to engineer a laser beam with much larger angular momentum, by having the rays skewed with respect to the propagation direction, as shown in Fig. 1 (b). The envelope of the ray form a hyperboloid, the same that can be obtained by attaching string to two circular basis, and twisting the two circles as the basis are pulled apart. The class of beams made of rays that twist around the axis of propagation has been call “Laguerre modes”, a different family than the “Gaussian modes”, usually emitted by a laser.o The linear momentum of the photon $h\nu$ has a component $h\nu \sin\theta$ in the plane normal to the propagation axis (which makes an angle $\theta$ with the light rays). The linear momentum times the arm $w_0$ (the radius of the beam at the waist) is an angular momentum, dubbed “orbital angular momentum”, considerably larger than the momentum due to the photon spin. For the example of a 20 kW laser, angle of $6^o$ and a beam radius of 1 mm, the kinetic energy of such a beam $Ww_0 \sin\theta/c$ corresponds to $7 \cdot 10^{-9} J$. The mechanical correspondent is our North pole experimentalist holding a 1.3 kg bird at arms length, rather than just one feather.

It is not conceivable that the kinetic energy associated with the rotation of a feather rotating about an axis at the rate of one turn/day would be measurable. The “precision of light” however is such that the faintest forces are perceived. The minuscule angular momentum of the photon was measured 24 years before the appearance of the first laser by Beth [1]. We will see that the polarization of light can be changed from linear to elliptical to circular with birefringent material, or waveplates. A half wave plate will change right circular polarization into left circular. In a similar manner as a mirror gets a recoil from the momentum of the light that bounces off it, the half wave plate having switched the direction of rotation of the electric field will experience by reaction a torsion. The torque applied by circular polarized light on a waveplate (half wave) that converts it from right circular to left circular was measured by suspending the waveplate on a thin wire, creating a very sensitive “torsion pendulum” [1].

# References

[1] R. A. Beth. Mechanical detection and measurement of the angular momentum of light. *Physical Review*, 50:115–125, 1936.