Homework 9: Happy Thanksgiving
due Thursday, November 29

1 Grating

You are asked to select a square flat grating 10 cm wide, for best wavelength selection (in first order) at 600 nm. The clearance is such that the acceptance angle for the incident beam ranges from -89^o to +89^o.

1. What is the configuration (angle of incidence and diffraction) and groove density that would give you the best resolution?
2. Assuming you can resolve a deviation angle of 1^o, what is the corresponding resolution in wavelength $\Delta\lambda_1$?
3. What is the diffracted angle (first order) at the second harmonic (300 nm) and the resolution $\Delta\lambda_2$ corresponding to a deviation angle of 1^o?
4. Can you achieve the same resolution at 300 nm (rather than for 600 nm) in higher order (if so, which order)?

2 Ring cavity

Consider the ring cavity sketched in Fig. 1. The ring uses mirrors of 100% reflectivity, and one mirror of field reflectivity $r = 99.99\%$ which serves as input/output to this resonator. We are operating around a wavelength of 800 nm. The perimeter of the ring is 60 cm. Assume the mirrors have no dispersion.

1. Find the transfer function for this ring resonator

2. If you send a beam into this resonator, how can you determine whether it is in condition of resonance or not?

3. Assume you send a train of ultrashort pulses through the resonator. What are the condition(s) for the laser (near 800 nm) to be on resonance with that cavity?

Solutions

2.1 Grating problem

1. The resolution is typically connected to the number of grooves that are illuminated. The smaller the groove, the larger the number of grooves over the dimension of the grating. In the grating equation:

$$\sin\alpha + \sin\beta = \frac{\lambda}{d}. \tag{1}$$

For a given λ, the smallest groove will correspond to the largest left hand side. The maximum value is $\sin\alpha = \sin\beta \approx 1$, i.e. the closest possible to grazing incidence (which is here 89^o). The optimum configuration is thus $d = \lambda/2$ in first order. This (extreme) Littrow configuration has been used for making very narrow bandwidth dye laser, and narrow line external cavity semiconductor lasers.

2. Taking the derivative of Eq. (1):

$$\cos\beta\Delta\beta = \frac{\Delta\lambda_1}{d} = 2\frac{\Delta\lambda_1}{\lambda}, \tag{2}$$

which leads to the resolution:

$$\Delta\lambda_1 = \frac{1}{2}\lambda \times \cos\beta \times \Delta\beta = 0.00015\lambda = 0.09\text{nm}$$

3. At the second harmonic, $\lambda = d$, hence $\sin\beta \approx 0$. Using Eq. 2) again:

$$\cos\beta\Delta\beta \approx \Delta\beta = \frac{\Delta\lambda_2}{d} = \frac{\Delta\lambda_1}{\lambda}, \tag{3}$$

which leaves us with the resolution:

$$\Delta\lambda_2 = \lambda \times \Delta\beta = 0.017\lambda \approx 5\text{nm}$$

4. The grating equation (1) in second order at 300 nm is identical to the one at 600 nm in first order. Therefore we get the same resolution ins second order at 300 nm than in first order at 600 nm (Littrow and grazing incidence).

2.2 Ring cavity

This problem is identical to the simplest version of the Gires Tournois interferometer. The total transmission of the device is always unity, since there is only one pathway for input and one for output (energy conservation). The complex transmission is:

$$R(\Omega)e^{-i\Psi(\Omega)} = \frac{-r + e^{i\delta}}{1 - re^{i\delta}} \tag{4}$$

with $r = 0.9999$, $|R(\Omega)| = 1$, and δ is the phase factor corresponding to a propagation through the perimeter.

2.2.1 If you send a beam into this resonator, how can you determine whether it is in condition of resonance or not?

There would be two possibilities to determine the resonance. The first one involves sampling the intracavity power by taking the Brewster angle reflection of the piece of glass inserted in the ring. A second method would be to monitor the duration of the pulse transmitted by the devise. The effective path of glass is $d \times (r+1)/(r-1)$ only in the condition of resonance.

2.2.2 Assume you send a train of ultrashort pulses through the resonator. What are the condition(s) for the laser (near 800 nm) to be on resonance with that cavity?

The ring cavity should satisfy the resonance condition for the phase (integer number of wavelengths in the cavity) and for the group velocity (the period of the train of pulses should exactly match the group delay of the pulse circulating in the cavity.