Electron Gas

An electromagnetic wave of frequency ω is incident on a region containing $N_e(t)$ electrons/cc.

Equation of motion

$$m_e \frac{dv}{dt} = -eE + e[v \times B] - m_e \nu_c v \tag{1}$$

Electron current For the driving field, we write $E = \mathcal{E} \exp(i\omega t)$. Let us choose an axis x along the E field, and z for the direction of propagation. The B field will induce a longitudinal (along z) component of the motion of the electron, which we neglect.

The main effect of the field will be to impart an electron motion along x, with a velocity v_x, giving rise to a current $N_e(t)ev_x$ given by:

$$N_e(t)ev = \frac{N_e e^2 \mathcal{E}}{m_e(\nu_c + i\omega)} \tag{2}$$

(We are considering here only linear polarization). In the case of elliptical or circular polarization, one has to add another (drift in the case of circular) component - see [**?**]. That drift component changes completely the response.

Dielectric constant To find the dielectric constant, we use Maxwell's equation:

$$\begin{aligned}\nabla \times H &= i\omega\epsilon_0\mathcal{E} + \frac{N_e e^2 \mathcal{E}}{m_e(\nu_c + i\omega)} \\ &= i\omega\epsilon_0\mathcal{E}\left[1 - \frac{N_e e^2}{m_e\epsilon_0(\omega^2+\nu_c^2)} - i\frac{N_e e^2}{m_e\epsilon_0(\omega^2+\nu_c^2)}\frac{\nu_c}{\omega}\right],\end{aligned} \tag{3}$$

where we have assumed that there is no magnetic field. One could also attribute the source term on the right of Eqs. (3) to a displacement current:

$$\nabla \times H = \frac{\partial D}{\partial t} = \frac{\partial \epsilon E}{\partial t} = \epsilon\frac{\partial E}{\partial t} + E\frac{\partial \epsilon}{\partial t} \tag{4}$$

where the last term $\frac{\partial \epsilon}{\partial t}$ is traditionally set to zero (only justified in a plasma at equilibrium).

In linear polarization, one can define an epsilon proportional to the electric field, in what is called the Drude model:

$$\begin{aligned}\epsilon &= \epsilon_0\left[1 - \frac{N_e e^2}{m\epsilon_0(\omega^2+\nu_c^2)} - i\frac{N_e e^2}{m_e\epsilon_0(\omega^2+\nu_c^2)}\frac{\nu_c}{\omega}\right] \\ &= \epsilon_0\left[1 - \frac{\omega_p^2}{\omega^2+\nu_c^2}\left(1 + i\frac{\nu_c}{\omega}\right)\right] \\ &= \epsilon_r + i\epsilon_i.\end{aligned} \tag{5}$$