Previously

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How to flip from single mode to multimode in a single mode fiber

Tapering: a changes from 4 µm to 5 µm, Δn from 0.0049 to 0.445

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Polarization rotation -- linear

in PM fibers. Temp sensor, power meter, stabilization

Today

Polarization rotation -- nonlinear

In nonlinear optics, 1 + 1 = 3! (????????)

Cross phase modulation

Mode-locked fiber lasers

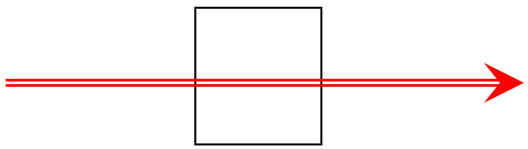
Velocities in fibers

Phase velocity, group velocity, velocity in gain/absorber

Polarization rotation -- nonlinear

In nonlinear optics, 1 + 1 = 3!   (????????)

Self

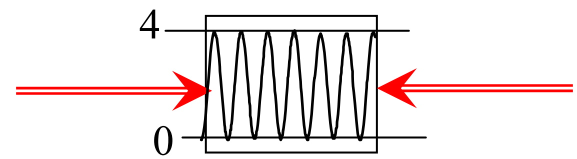


$$\boxed{\frac{dI}{dz} = \frac{\alpha_0 I}{1+\frac{I}{I_s}}}$$

$$\frac{d|E|^2}{dz} = \frac{\alpha_0 |E|^2}{1+\frac{|E|^2}{\mathcal{E}_s^2}}$$

Saturation

Colliding



$$\begin{aligned} |E|^2 &= \mathcal{E}^2\left(e^{2ikz} + e^{-2ikz} + 2\right) \\ &= 2\mathcal{E}^2(\cos 2kz + 1) \end{aligned}$$

$$2\frac{d|E|^2}{dz} \approx \left(\alpha_0 |E|^2\right)\left(1 - \frac{|E|^2}{\mathcal{E}_s^2}\right)$$

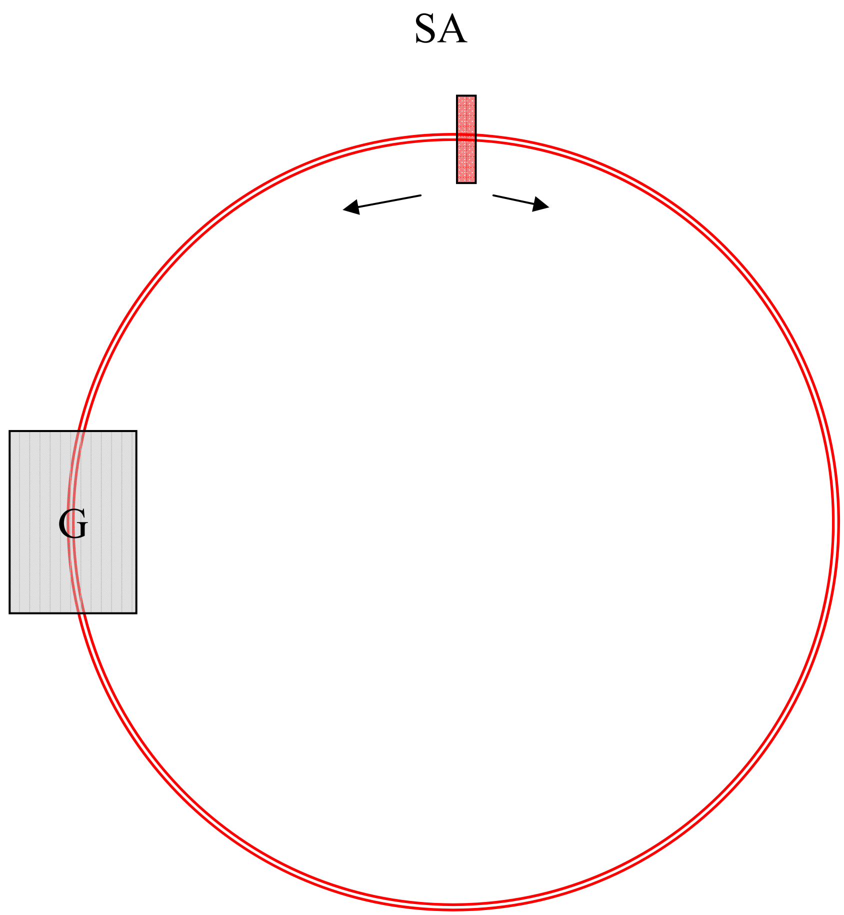
$$2\frac{d|E|^2}{dz} \approx 2\alpha_0\mathcal{E}^2\left\langle(1+\cos 2kz) - 2\frac{\mathcal{E}^2}{\mathcal{E}_s^2}(1+\cos 2kz)^2\right\rangle$$

$$2\frac{d|E|^2}{dz} \approx 2\alpha_0\mathcal{E}^2\left\langle(1+\cancel{\cos 2kz}) - 2\frac{\mathcal{E}^2}{\mathcal{E}_s^2}(1+\frac{\cancel{\cos 4kz}+1}{2} + 2\cancel{\cos 2kz})\right\rangle$$

$$2\frac{d|E|^2}{dz} \approx 2\alpha_0\mathcal{E}^2\left[1 - 2\frac{\mathcal{E}^2}{\mathcal{E}_s^2}(1+\tfrac{1}{2})\right]$$

$$\boxed{\frac{d|E|^2}{dz} = \frac{\alpha_0\mathcal{E}^2}{1+3\frac{\mathcal{E}^2}{\mathcal{E}_s^2}}}$$

Application: Colliding pulse mode-locking



Cross phase modulation

The nonlinear index (Kerr effect) comes from: $P_{NL} = \epsilon \chi^{(3)} \mathbf{EEE}$

Same *k*, different frequencies, or same frequency, different *k*. $E = \mathcal{E}_1 e^{i\omega_1 t} + \mathcal{E}_2 e^{i\omega_2 t}$

$$P_{NL} \propto \left(\mathcal{E}_1 \mathcal{E}_1^* + \mathcal{E}_2 \mathcal{E}_2^*\right) \mathcal{E}_1 e^{i\omega_1 t} + \ldots + \left(\mathcal{E}_1 \mathcal{E}_2^* e^{i(\omega_1 - \omega_2)t}\right) \mathcal{E}_2 e^{i\omega_2 t}$$

$$P_{NL}(\omega_j) \propto \chi^{(3)} \left(|\mathcal{E}_1|^2 + 2|\mathcal{E}_2|^2\right) \mathcal{E}_1 e^{i\omega_1 t}$$

$$\Delta n_j = n_2 |E_j|^2 + 2|E_{3-j}|^2$$

1	1	2
2	2	1

Co-propagating beams in fibers:

$$\Delta \phi_{NL} = k_0 \Delta n \ell$$

Polarization rotation

The nonlinear index (Kerr effect) comes from: $P_{NL} = \epsilon \chi^{(3)} \mathbf{EEE}$

Same *k*, frequency, orthogonal polarization. $E = \mathcal{E}_x e^{i\omega t} + \mathcal{E}_y e^{i\omega t}$

$$P_{NL} \propto \epsilon_0 \chi \sum_j \left[E_i E_j E_j^* + E_j E_i E_j^* + E_j E_j E_i^*\right]$$

$$\begin{aligned} P_x &= \epsilon_0 \chi \sum_j \left[E_x E_y E_y^* + E_y E_x E_y^* + E_y E_y E_y^* \right. \\ &\quad \left. + E_x E_x E_x^* + E_x E_x E_x^* + E_x E_x E_x^*\right] \end{aligned}$$

$$P_x = \epsilon_0 \chi \sum_j \left[3|E_x|^2 + 2|E_y|^2\right] E_x$$

$$P_x = 3\epsilon_0 \chi \sum_j \left[|E_x|^2 + \frac{2}{3}|E_y|^2\right] E_x$$

$$\begin{aligned}\Delta n_{\mathrm{nl},x} &= n_2\left[|\tilde{\mathcal{E}}_{0x}|^2 + \frac{2}{3}|\tilde{\mathcal{E}}_{0y}|^2\right]\\ \Delta n_{\mathrm{nl},y} &= n_2\left[|\tilde{\mathcal{E}}_{0y}|^2 + \frac{2}{3}|\tilde{\mathcal{E}}_{0x}|^2\right].\end{aligned}$$

In an element of thickness d_m, this induced birefringence leads to a phase change between the x and y components of the field vector

$$\Delta\Phi(t) = \frac{2\pi}{\lambda_\ell}\left(\Delta n_{\mathrm{nl},x} - \Delta n_{\mathrm{nl},y}\right) = \frac{2\pi n_2 d_m}{3\lambda_\ell}\left[|\tilde{\mathcal{E}}_{0x}(t)|^2 - |\tilde{\mathcal{E}}_{0y}(t)|^2.\right]$$

The phase shift is time dependent, and, in combination with another element, can represent an intensity-dependent loss element. Consider a sequence of such birefringent element and a linear polarizer. The incident pulse is

$$\begin{aligned}\mathcal{E}_{0x}(t) &= \mathcal{E}_0(t)\cos\alpha\\ \mathcal{E}_{0y}(t) &= \mathcal{E}_0(t)\sin\alpha\end{aligned}$$

The pass direction of the polarizer is at $\alpha + 90^o$ resulting in zero transmission through the sequence for low-intensity light ($\Delta\Phi \approx 0$). Neglecting a common phase the field components at the output of the nonlinear element are

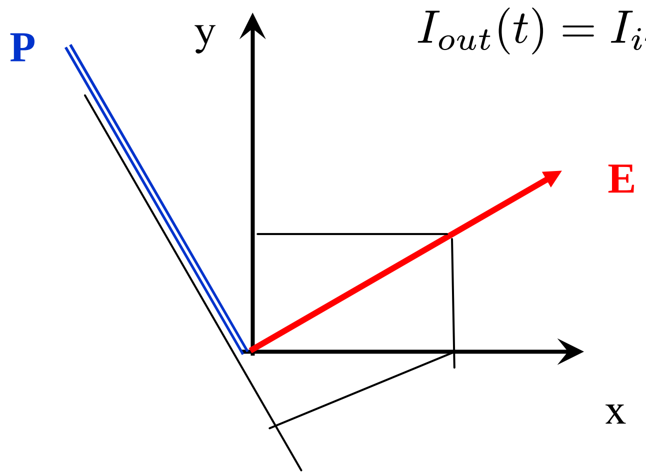
$$\begin{aligned}\mathcal{E}_x'(t) &= [\mathcal{E}_0(t)\cos\alpha]\cos(\omega_\ell t)\\ \mathcal{E}_y'(t) &= [\mathcal{E}_0(t)\sin\alpha]\cos[\omega_\ell t + \Delta\Phi(t)]\,.\end{aligned}$$

Next the pulse passes through the linear polarizer. The total transmitted field is the sum of the components from $\mathcal{E}_x'(t)$ and $\mathcal{E}_y'(t)$ along the polarizer's path direction

$$\mathcal{E}_{out}(t) = \mathcal{E}_0(t)\cos\alpha\sin\alpha\,\{-\cos(\omega_\ell t) + \cos[\omega_\ell t + \Delta\Phi(t)]\}$$

The total output intensity $I_{out}(t) = \langle\mathcal{E}^2(t)\rangle$ is

$$I_{out}(t) = I_{in}(t)\frac{1}{2}[1 - \cos\Delta\Phi(t)]\sin^2(2\alpha).$$



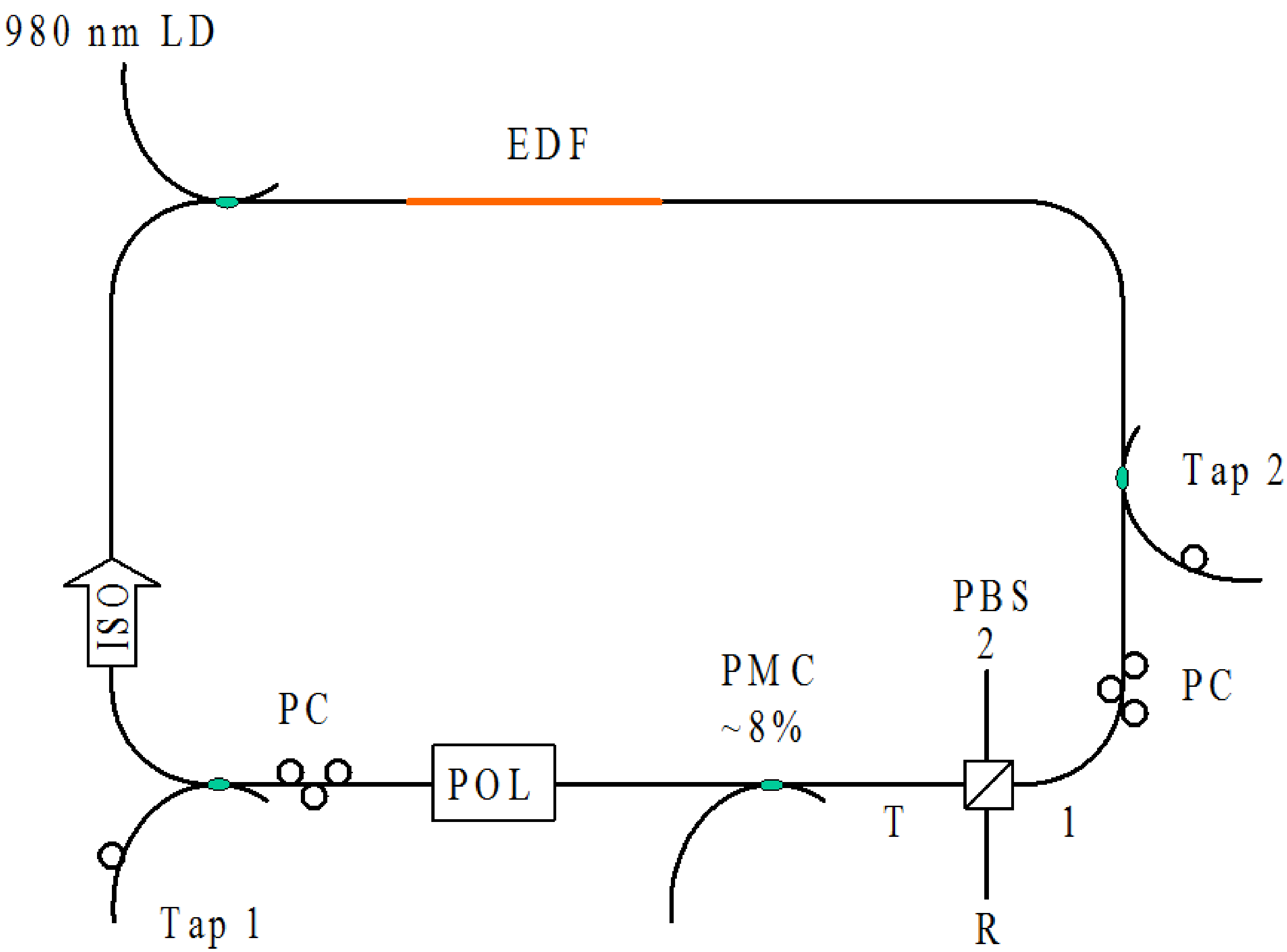
Let us now assume a Gaussian input pulse $I_{in} = I_0 \exp\left[-2(t/\tau_G)^2\right]$ and parameters of the nonlinear element so that for the pulse center the phase difference

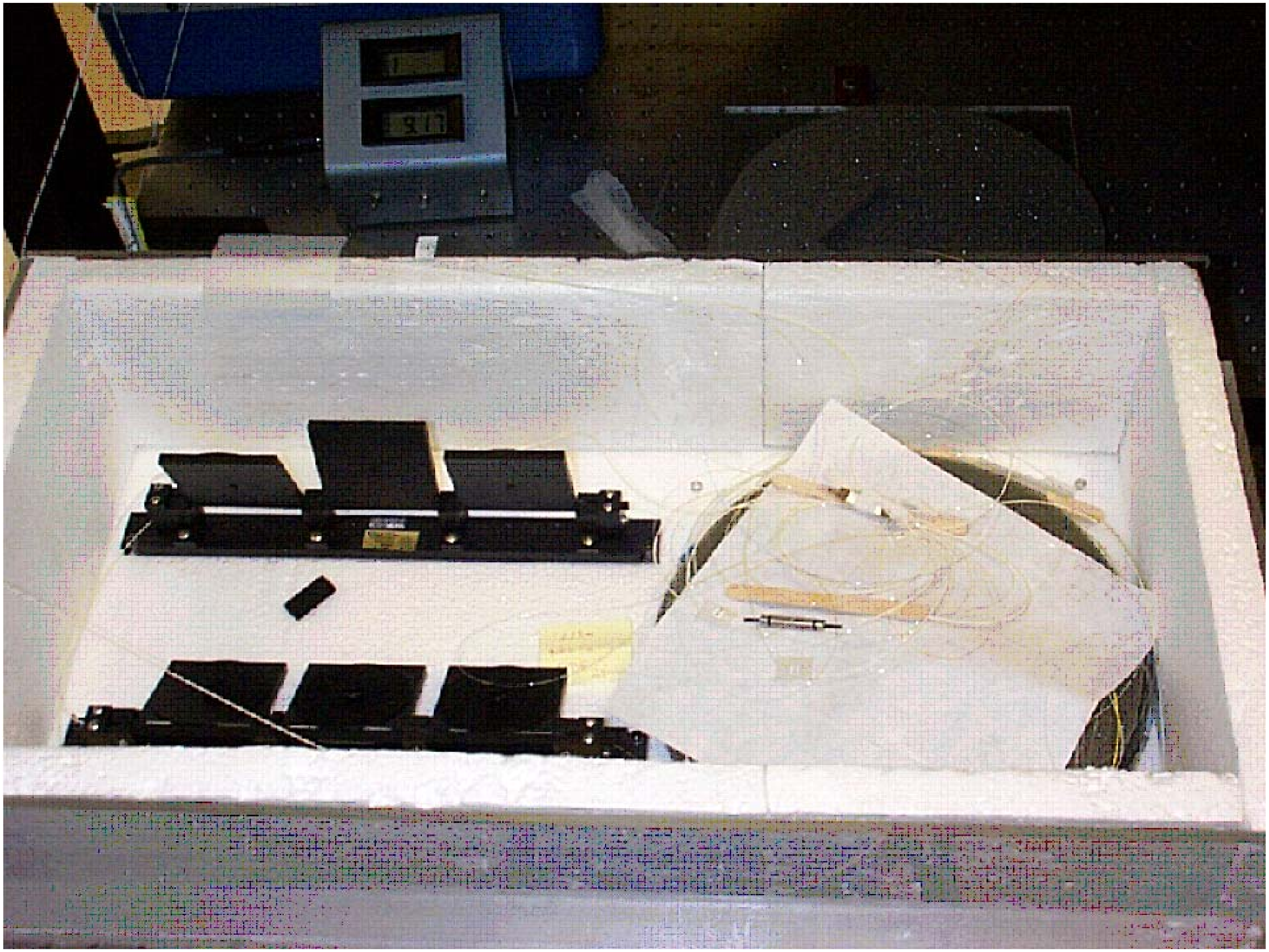
$$\Delta\Phi(t=0) = \frac{2\pi n_2 d_m}{3\lambda_\ell} \mathcal{E}_0^2(t=0)\left(\sin^2\alpha - \cos^2\alpha\right) = \pi.$$

For this situation we obtain a transmitted pulse

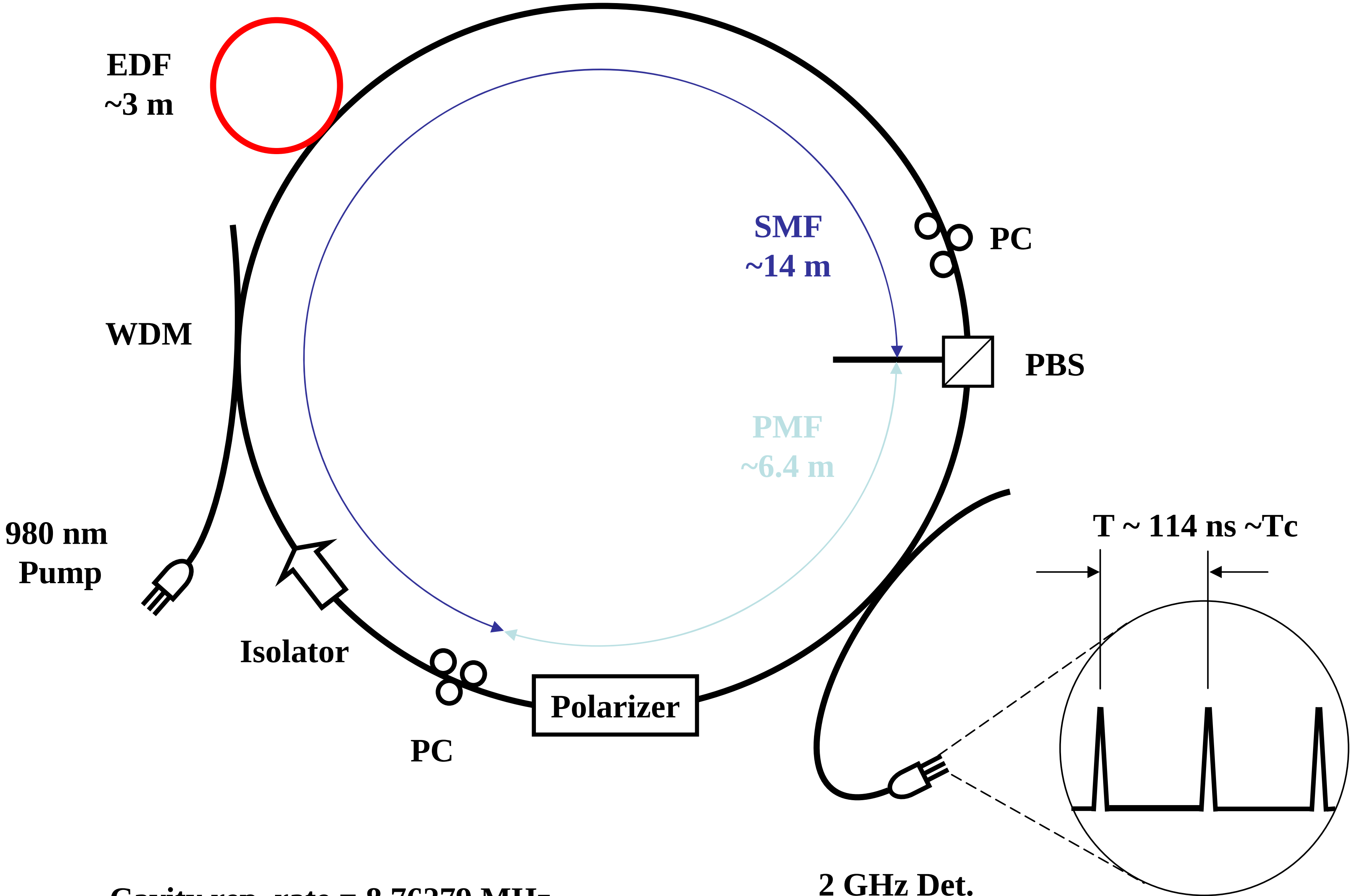
$$I_{out}(t) = \frac{1}{2} I_{in}(t)\left\{1 - \cos\left[\pi e^{-2(t/\tau_G)^2}\right]\right\}.$$

The transmission is maximum (= 1) where the nonlinear element acts like a half-wave plate that rotates the polarization by 90^o, lining it up with the pass direction of the polarizer. For the parameters chosen here this happens at the pulse center (t = 0). The phase shift $\Delta\Phi$ is smaller away from the pulse center producing elliptically polarized output and an overall transmission that approaches zero in the pulse wings. Thus this sequence of elements can give rise to an intensity dependent transmission similar to a fast absorber.


980 nm LD
EDF
Tap 2
ISO
PBS
2
PMC
~8%
PC
PC
POL
T
1
R
Tap 1



The question of how to start the mode locking process is solved in several ways. Experimentally, the Polarization Controllers (PC) have the greatest effect, given sufficient gain in the cavity. By manually adjusting the 6 paddles, we can sometimes get lucky and find CWML pulses on the oscilloscope. In practice, looking at the time trace does not get us close enough. It is easy to miss the ML'ing by moving the PC too fast, or not fast enough, etc.. We can monitor the optical spectrum and see the broadening of the bandwidth in the output. The other method of starting the CWML is to find the ``sweet spots'' on the box or the table and hitting those spots.



Cavity rep. rate = 8.76279 MHz

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Cross phase nodulation

Mode-locked fiber lasers

Velocities in fibers

Phase velocity, group velocity, velocity in gain/absorber

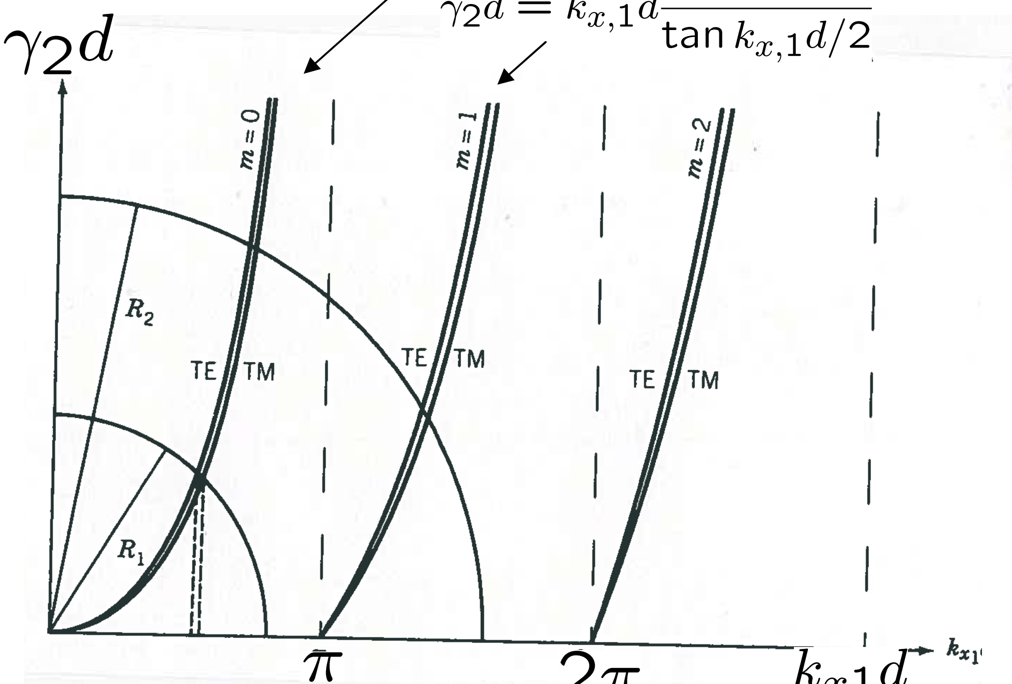
Slab waveguide solution

$$\gamma_2 d = k_{x,1} d \tan k_{x,1} d/2$$

Evanescent wave

$$\gamma_2 d = k_{x,1} d \frac{1}{\tan k_{x,1} d/2}$$

$\gamma_2 d$



π 2π $k_{x1} d$

$$\boxed{R = k_0 d \sqrt{n_1^2 - n_2^2}}$$

Transverse k vector =

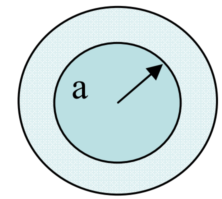
$$n_1 k_0 \cos\theta_1$$

In metallic waveguides

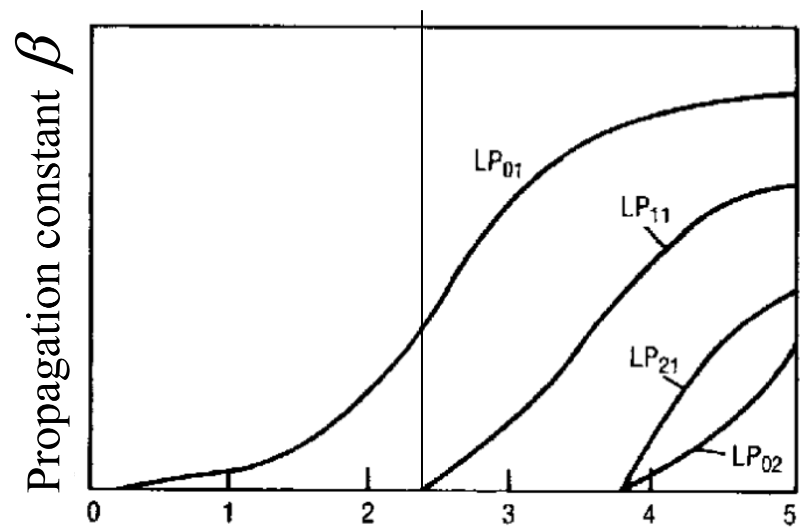
Phase velocity: $v_p = \dfrac{\Omega}{\beta} = \dfrac{\Omega}{k \sin\theta_m}$

Group velocity: $\dfrac{1}{v_g} = \dfrac{d\beta}{d\Omega} \quad v_g = \dfrac{c}{n}\sin\theta_m$

Velocities in fiber?



V = 2.405

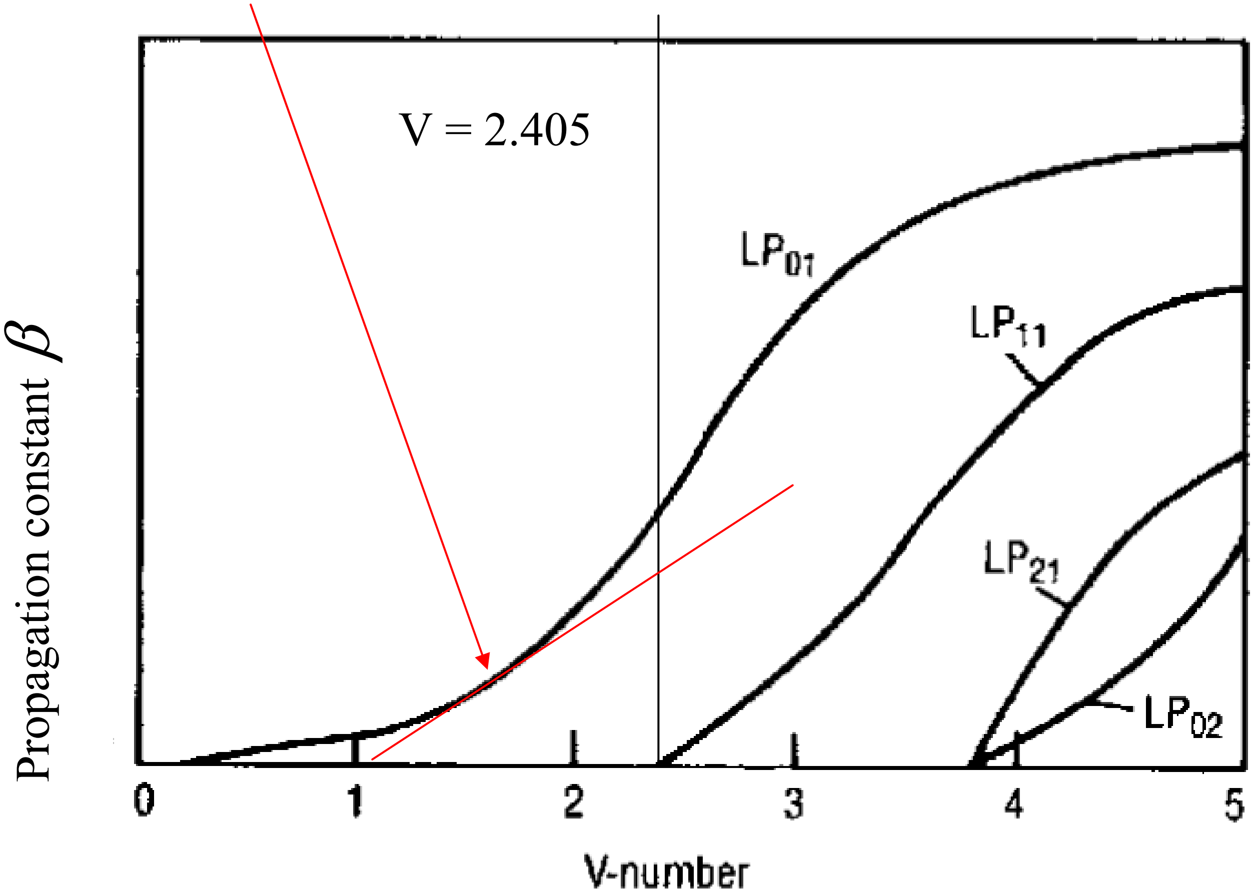


$$V = k_0 a \sqrt{n_1^2 - n_2^2}$$

$$V = k_0 a\sqrt{n_1^2 - n_2^2} = \frac{\Omega}{c} a\sqrt{n_1^2 - n_2^2}$$

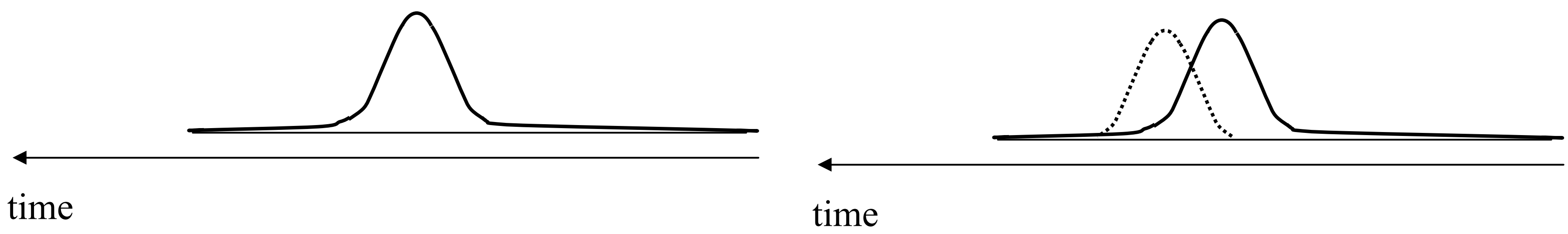
Phase velocity: $\dfrac{\Omega}{\beta} = \dfrac{V}{\beta}\dfrac{1}{a\sqrt{n_1^2 - n_2^2}}$

Group velocity: $\dfrac{1}{v_g} = \dfrac{d\beta}{d\Omega}$

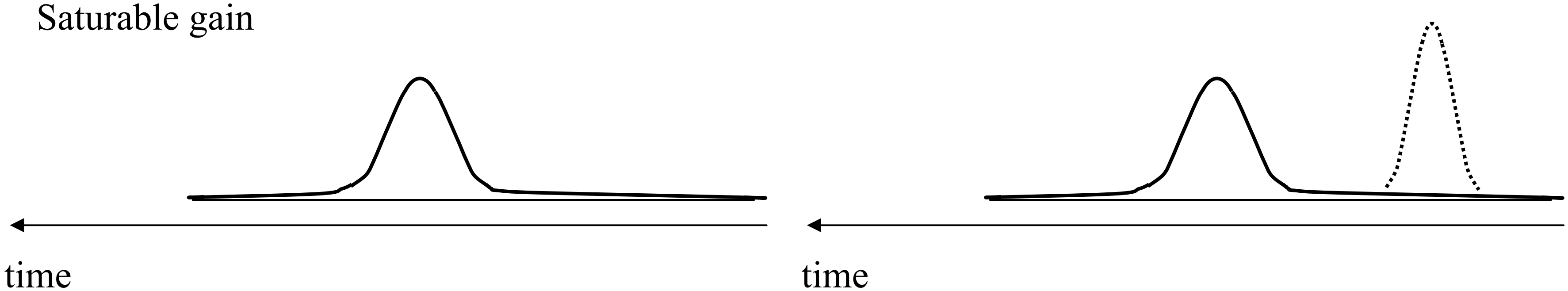


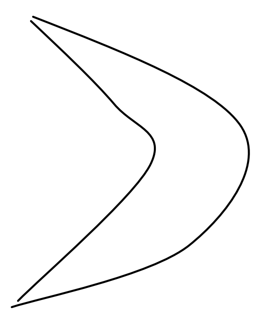
GROUP VELOCITY APPLIES ONLY TO LINEAR DIELECTRICS

Saturable absorber



Saturable gain





SOVIET PHYSICS JETP VOLUME 23, NUMBER 1 JULY, 1966

NONLINEAR AMPLIFICATION OF LIGHT PULSES

N. G. BASOV, R. V. AMBARTSUMYAN, V. S. ZUEV, P. G. KRYUKOV, and V. S. LETOKHOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

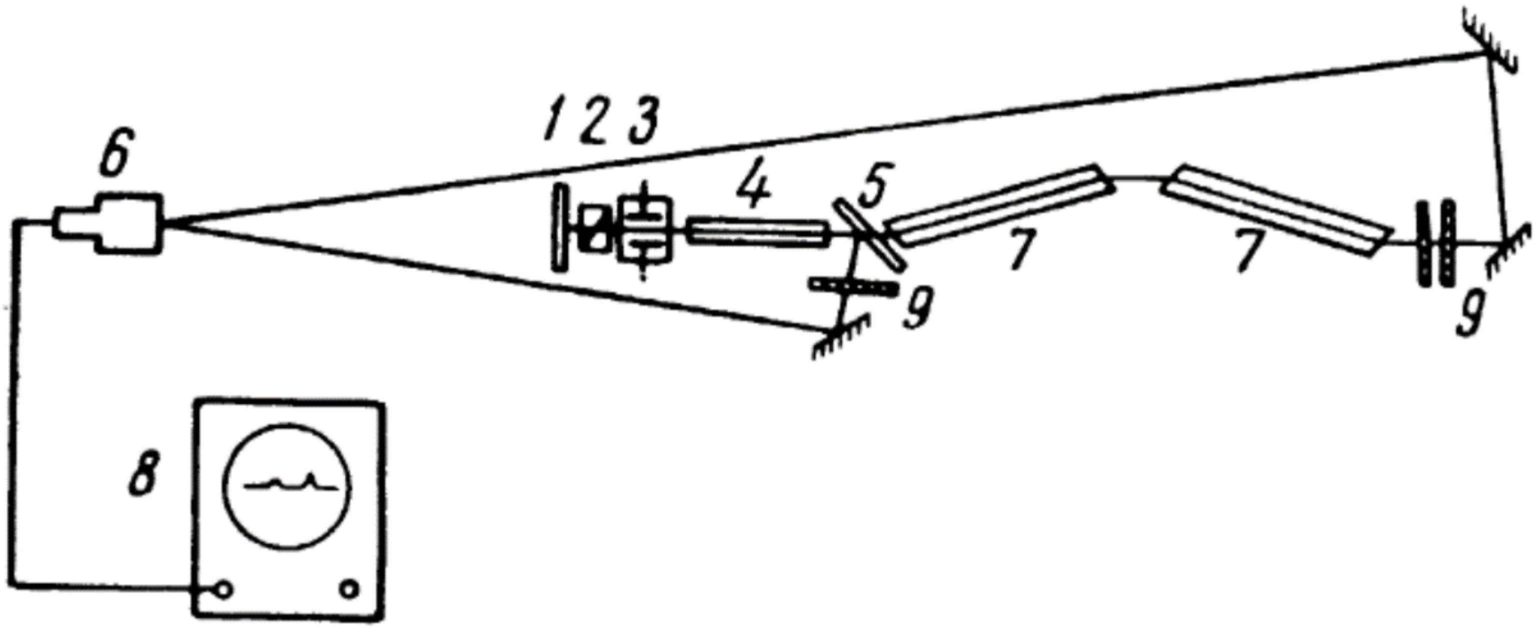


FIG. 5. Diagram of setup: 1 – laser mirror, 2 – polarizer, 3 – Kerr cell, 4 – laser ruby crystal, 5 – glass plate, 6 – coaxial photocell, 7 – amplifier ruby crystal, 8 – SI-14 oscilloscope, 9 – neutral light filters. The input mirror of the laser is at the end of crystal 4.

The velocity of a pulse in a **saturable gain medium**

$$\frac{v}{c} = 1 + \frac{c\tau_p}{2}(\alpha - \gamma)$$

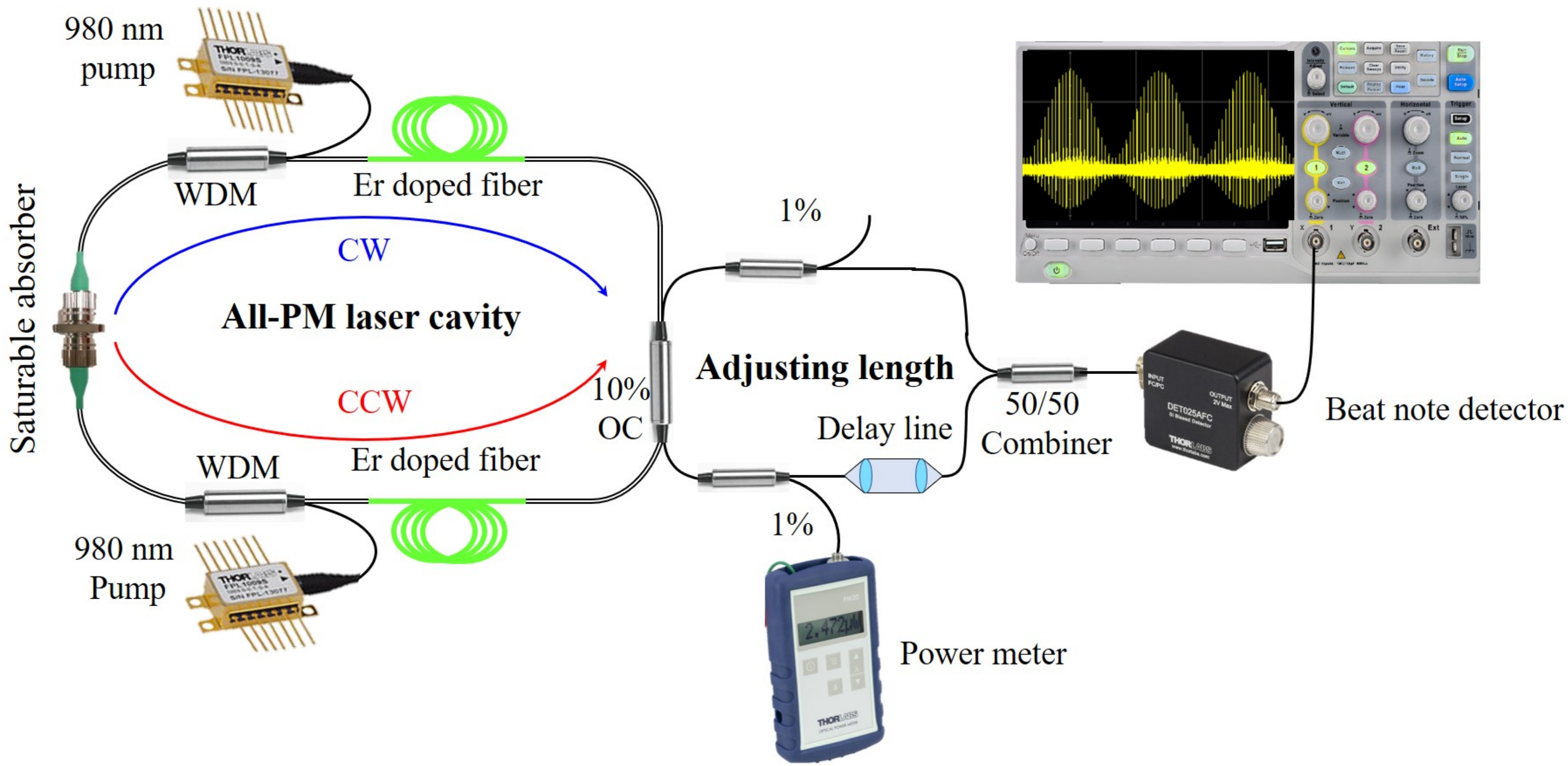
c	Speed of light in the gain
τ_p	Pulse duration
α	Small signal gain coefficient (per unit length)
γ	Loss coefficient (per unit length)

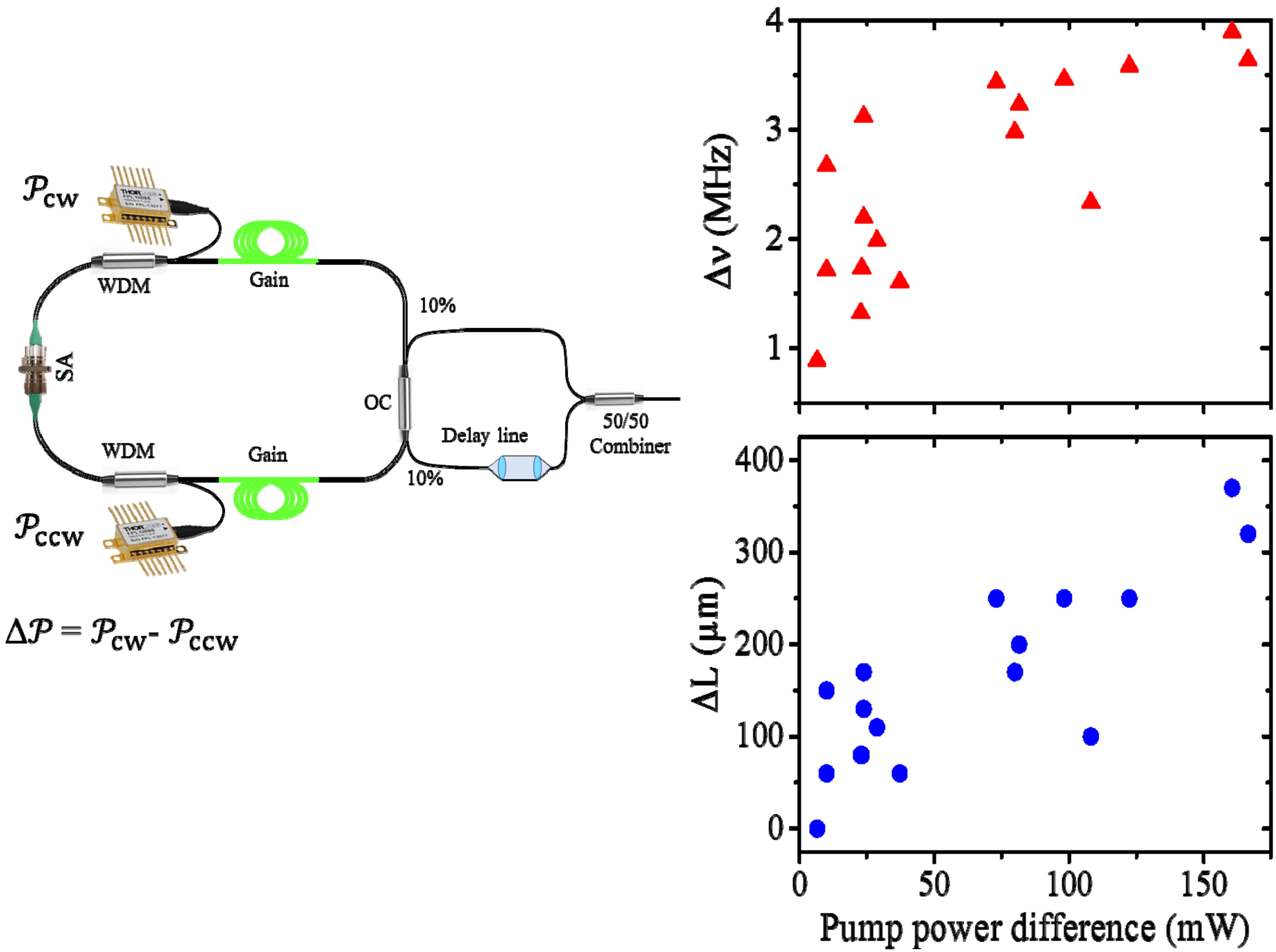
The velocity of a pulse in a **ring fiber laser**

$$\frac{v}{c} = 1 + \frac{c\tau_p}{2}\left(\alpha_0 \frac{\mathcal{P}}{\mathcal{P}_0} - \gamma\right)$$

c	Speed of light in the gain	τ_p	Pulse duration	γ	Loss coefficient (per unit length)
α	Small signal gain coefficient (per unit length)			$\mathcal{P}$	Pump power
α_0	Small signal gain at the laser threshold			$\mathcal{P}_0$	Pump power at the laser threshold

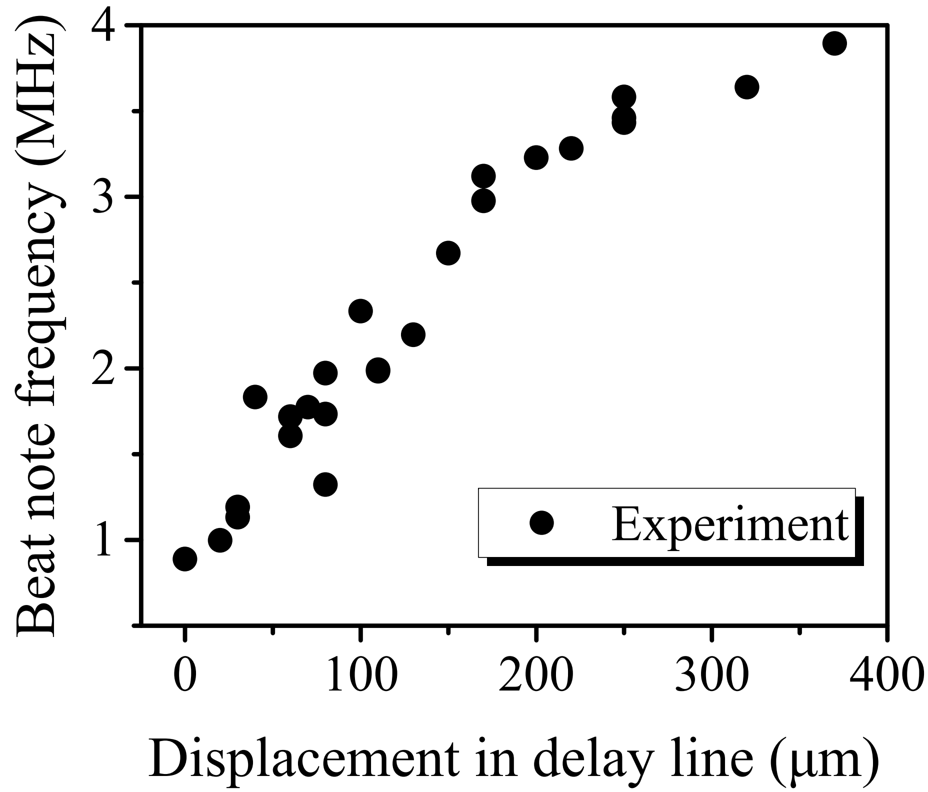
Example of pulse velocity >> *c* in a fiber laser

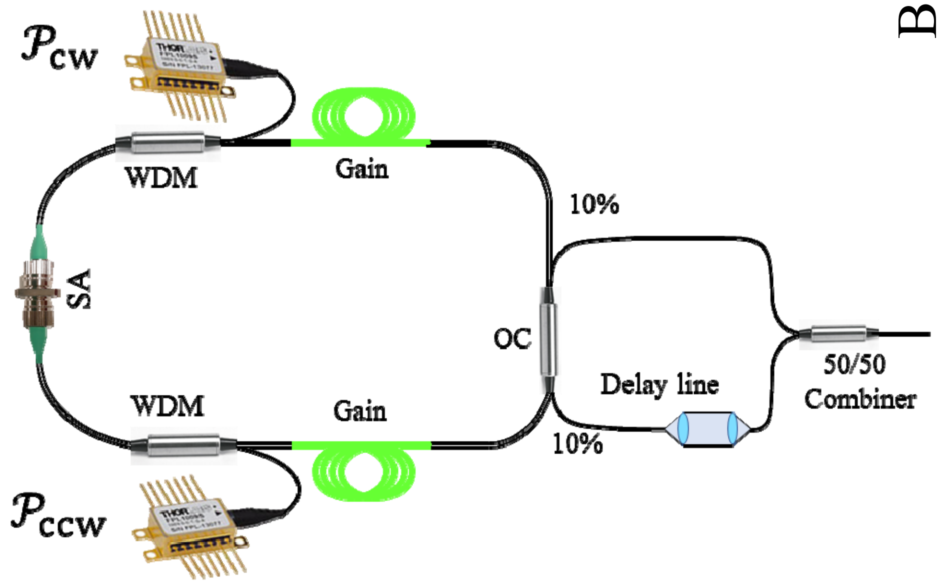




$\mathcal{P}_{cw}$
WDM
Gain
10%
SA
OC
Delay line
50/50
Combiner
WDM
Gain
10%
$\mathcal{P}_{ccw}$
$\Delta\mathcal{P} = \mathcal{P}_{cw} - \mathcal{P}_{ccw}$
Δν (MHz)
ΔL (μm)
Pump power difference (mW)
0
50
100
150
1
2
3
4
0
100
200
300
400

Beat note frequency vs displacement in the delay line





$$\Delta\mathcal{P} = \mathcal{P}_{cw} - \mathcal{P}_{ccw}$$

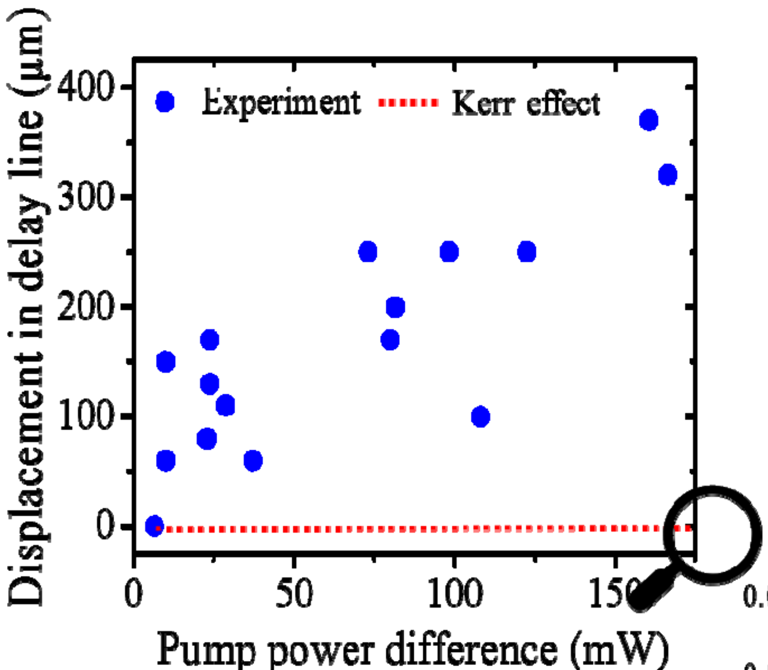
$$n_2 = 3 \times 10^{-16}\ \frac{cm^2}{W}$$

$$I_{cw}=1.2\ \frac{GW}{cm^2} \quad ,L_{cw}= 2765\ mm$$

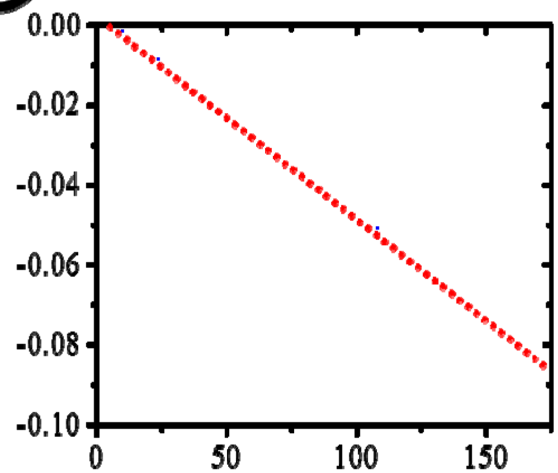
$$I_{cw}=0.5\ \frac{GW}{cm^2} \quad L_{ccw}= 2475\ mm$$

$$\Delta L = n_2 (I_{cw} L_{cw} - I_{ccw} L_{ccw})$$

ΔL=0.6 μm



Displacement in delay line due to the **Kerr effect**



Experiment

ΔL =260 μm

The velocity of a pulse in a **saturable gain medium**

$$\frac{v}{c} = 1 + \frac{c\tau_p}{2}(\alpha - \gamma)$$

c	Speed of light in the gain
τ_p	Pulse duration
α	Small signal gain coefficient (per unit length)
γ	Loss coefficient (per unit length)

The velocity of a pulse in a **ring fiber laser**

$$\frac{v}{c} = 1 + \frac{c\tau_p}{2}\left(\alpha_0 \frac{\mathcal{P}}{\mathcal{P}_0} - \gamma\right)$$

c	Speed of light in the gain	τ_p	Pulse duration	γ	Loss coefficient (per unit length)
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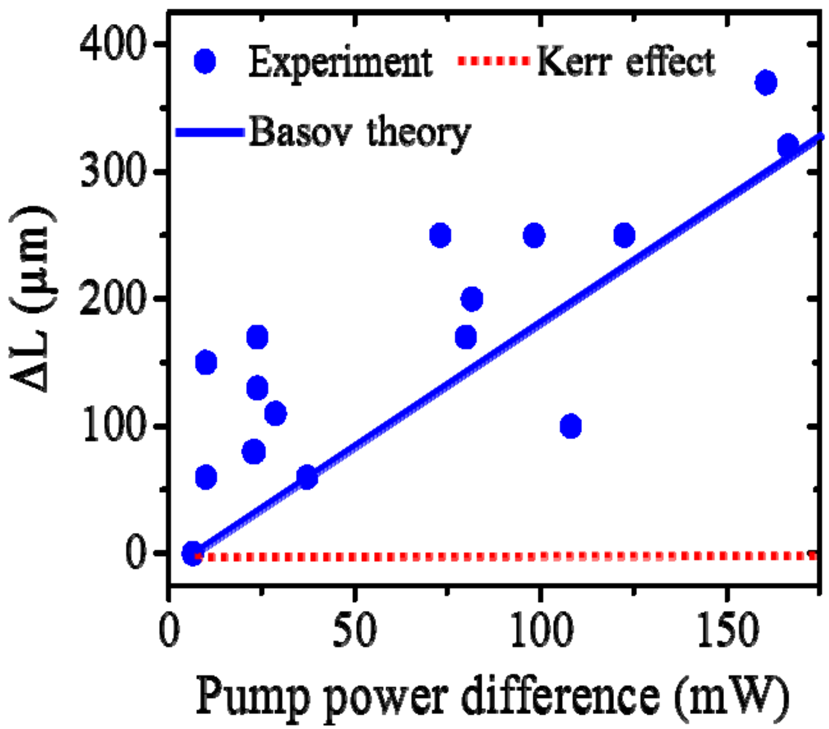

Parenthesis: the frequency comb