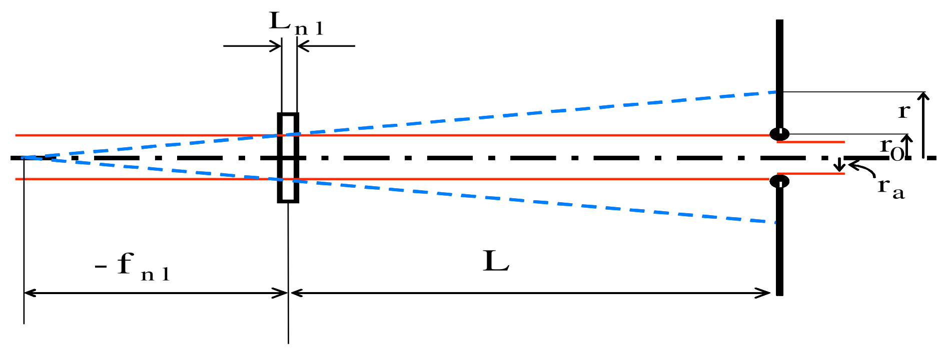

Figure E.1: A simplified power limiter, including a nonlinear lens acting on a collimated beam of radius r_0, and an aperture of radius r_a. At high power, the nonlinear lens acquires a focal length of f_{nl}. The example shown refers to a negative nonlinearity leading to defocusing.

which may lead to pulse broadening and limits the overall energy transmission.

Example of a nonlinear cavity and Gaussian beam analysis

We proceed next to a numerical example of nonlinear lensing in a cavity using the Gaussian beam analysis of Section 5.5.3. Figure E.2 shows a ring cavity and the equivalent unit cell of the unfolded (linear) cavity. The cavity contains two identical focusing elements, a nonlinear lensing element, and an aperture. We choose the distance between L_1 and L_2, $2d_1 = 52.632$ mm, and $f = 25$ mm, and the wavelength of the radiation $\lambda = 1$ μm. The maximum length (stability limit) of the cavity segment between L_1 and L_2 that contains the aperture, is such that the image point of the waist close to the nonlinear element is at a distance d_1' of either lens given by $1/d_1' = 1/f - 1/d_1$. It is convenient to define the distance L between L_1 and L_2 with reference to the stability limit, i.e. $L = 2d_1' - \ell$ or

$$L = 2\frac{d_1 f}{d_1 - f} - \ell, \tag{E.2}$$

where ℓ is a positive length between 0 (stability limit) and $2d_1'$. We first determine the system matrix of this unit cell, $\mathcal{M}_1$, starting from the position of the NLE:

$$\mathcal{M}_1 = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \tag{E.3}$$

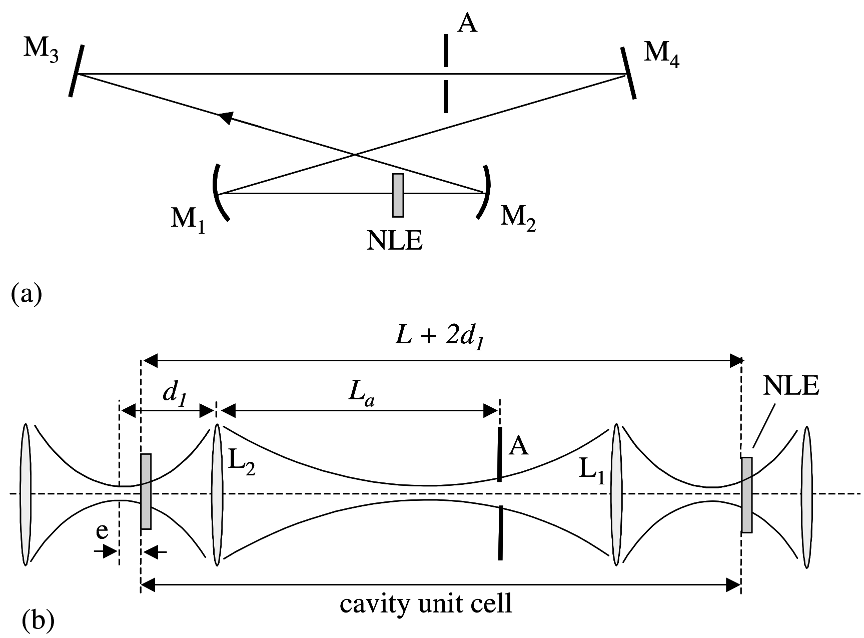

Figure E.2: (a) Ring cavity with a nonlinear lensing element NLE, two identical focusing mirrors (M_1 and M_2) with focal length f, two flat mirrors (M_3 and M_4), and an aperture A. (b) Unit cell of the equivalent unfolded (linear) cavity. For symmetry reasons two beam waists are formed, on either side, halfway in between lenses (mirrors) L_1 and L_2, that is, a distance d_1 and $L/2$ from lens L_1.

where the elements of the matrix are:

$$
\begin{aligned}
A_1 &= 1 + \frac{2d_1}{f} + \frac{2d_1 e}{f(d-f)} + \frac{\ell}{f} - \frac{(d_1+e)(2f+\ell)}{f^2} = 2.324 \\
B_1 &= \left\{ \left[\frac{2d_1 f}{d_1 - f} \right] - \ell \right\} \left[\left(1 - \frac{d_1}{f} \right)^2 - \frac{e^2}{f^2} \right] + 2d_1 - 2\frac{d_1^2 - e^2}{f} = -1.58 \text{ mm} \\
C_1 &= -\frac{2}{f} + \frac{2d_1}{f(d_1 - f} - \frac{\ell}{f^2} = 1.44 \text{ mm}^{-1} \\
D_1 &= 1 + \frac{2e+\ell}{f} - \frac{2d_1 e}{f(d_1 - f)} - \frac{(d_1 - e)\ell}{f^2} = -0.545.
\end{aligned}
\tag{E.4}
$$

The numerical values correspond to $e = 1$ mm and $\ell = 50$ mm.

The stability criterium of the cavity takes the simple form:

$$
\left| \frac{A_1 + D_1}{2} \right| = 1 - \frac{(d_1 - f)\ell}{f^2} < 1. \tag{E.5}
$$

The limit $\ell = 0$ marks a stability limit of the cavity, corresponding to a concentric type cavity with a beam waist $w_0 \to 0$. The parameter d_1 determines the length of the cavity [perimeter equal to $2d_1^2/(d_1 - f)$]. To obtain a stable cavity we choose a cavity perimeter shorter by the amount ℓ (50 mm in our numerical example).

The eigenvalue of the system matrix is $\tilde{s}_1 = (0.909 - 0.2895\ i)\ \text{mm}^{-1}$, to which corresponds a spot size of $w_1 = 33\ \mu$m, and a radius of curvature of $R_1 = 1.1$ mm (at the position of the NLE), as given by the Eq. (5.118). We note that the nonlinear crystal (NLE) is outside the Rayleigh range of the beam waist, because propagation by -1 mm shows a beam waist of about 7 μm.

The matrix for translating from the crystal to the aperture located at a distance L_a from L_2 is:

$$\begin{pmatrix} A_m & B_m \\ C_m & D_m \end{pmatrix} = \begin{pmatrix} 1 & L_a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 - e \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{L_a}{f} & (d_1 - e) - \frac{L_a}{f}(d_1 - f - e) \\ -\frac{1}{f} & -\frac{d_1 - f - e}{f} \end{pmatrix} \tag{E.6}$$

If we choose as distance to the aperture from L_2 a length $L_a = 300$ mm, we find for the matrix:

$$\begin{pmatrix} A_m & B_m \\ C_m & D_m \end{pmatrix} = \begin{pmatrix} -11 & 21.52\ \text{mm} \\ -0.04\ \text{mm}^{-1} & -0.01264 \end{pmatrix} \tag{E.7}$$

The complex beam parameter at the location of the aperture, in the absence of nonlinear lensing, is thus:

$$\tilde{s}_m = \frac{C_m + D_m \tilde{s}_1}{A_m + B_m \tilde{s}_1} = (-0.0336 - 0.00258)\ \text{mm}^{-1}, \tag{E.8}$$

which corresponds to a beam parameter of $w_m = 350\ \mu$m. Let us first look at the change in beam size induced by the nonlinear lensing at the location of the NLE, as given by Eq. (5.122). We find that

$$\delta\tilde{s} = \frac{1}{f_{\text{nl}}}(0.4521 + 0.6458i)\,. \tag{E.9}$$

We assume nonlinear lensing that produces a lens of focal length $f_{nl} = 500$ mm, which gives for the change in the complex s parameter $\delta s = 0.0009 + 0.00129i$. From that, the relative change in beam waist $\delta w_1/w_1 \sim 0.5 \times 0.00129/0.289$ is about 0.2%.

Application of Eq. (5.127) yields the change in complex beam parameter at the location of the aperture:

$$\delta\tilde{s}_m = \frac{1}{f_{nl}}\left[\frac{(0.4521+0.6458i)}{(A_m+B_m\tilde{s}_1)^2}\right] = \frac{1}{f_{nl}}\left[-0.0042-0.0056i\right]. \qquad \text{(E.10)}$$

The relative change in beam waist at the aperture, for $f_{\mathrm{nl}} = 500$, is $\delta w_m/w_m = 0.5 \times 0.0056/(500 \times 0.00258)$ and is about 0.2%.

The location of the aperture should be away from a beam waist. If we chose for instance $L_a = [2d_1 f/(d_1 - f) - \ell]/2 = 475$ mm, which brings us close to the second beam waist of the cavity, we find $\tilde{s}_m = 0.001 - 0.16i$, corresponding to a beam waist of 45 μm, and $\delta\tilde{s}_m = 0.017(1+i)/f_{\mathrm{nl}}$. The relative beam waist change $\delta w_m/(w_m)$ is only 0.01%.