Optics 463 — Homework 9 -Fraunhofer
due Tuesday, November 13, 2018

1 Diffraction of a Gaussian beam

1.1 Fraunhofer approximation

Consider a plane wave with a Gaussian intensity distribution of waist w_0 (as could be obtained from an infinite uniform plane wave, normally incident on an apodized aperture, or simply a Gaussian beam emerging from the flat output coupler of a laser). Calculate the diffracted field $\mathcal{E}(x, z)$ using the Fraunhofer approximation, considering only one transverse coordinate to find the radial distribution of the electrical field amplitude. Compare the results with that of Gaussian beam propagation, and explain the differences/similarities.

1.2 Paraxial approximation

Same as above, but using the paraxial approximation.

2 Telescope array

This problem involves the principle of reciprocity in Optics, and Fraunhofer diffraction. The purpose of a telescope array is to achieve higher resolution. We will make here the approximation that we have a single-dimensional problem. The telescope could be optical or RF. Let us consider an array made of antennas of 2 m long. The antennas are disposed symmetrically. One is at the center. The next two are 10 meter away (distance center to center). The last two are at 30 meter from the central one (center to center). We thus have a total of 5 antennas. The frequency of the radiation is 3 GHz.

1. Considering only the central antenna as a square aperture, find the corresponding angular resolution (of the telescope having only one antenna). (consider this to be a Fraunhofer diffraction pattern problem)
2. Give your best numerical estimate of the resolution of the array consisting of 5 antennas.
3. How will the result of the previous two parts be changed if the radiation is $3 \times 10^{14} Hz$?