Optics 463 — Homework for Monday, August 26, 2013

Spacecraft

In the 70's at the peak of excitement about space exploration, a propulsion scheme was proposed to visit another galaxy. Since it is extremely costly to bring up fuel to a spacecraft, it was proposed to attach huge reflectors to the spacecraft, and to "push" it with a powerful light beam. Another approach is to "beam up" energy with a powerful laser, which is then collected by solar panels, and converted into energy to power an ionic engine to propel the satellite or spacecraft.

1. To get a comparison between the two approaches, assume that the energy conversion is 10% efficient (from laser light to propulsion). Assume a continuous laser beam of 1 MW power, being completely collected by solar panels of 100 m diameter, and applied for 200 seconds. Assuming a mass of 1000 kg, what will be the velocity increase of the spacecraft after 200 s or irradiation?

2. For comparison, replace the solar panels by perfectly reflecting mirrors. The laser beam is now used to push the spacecraft by radiation pressure. What will be the velocity after 200 seconds (still a 1 MW laser beam)?

Note that the authors of the article had also an answer to the question: "how do you stop the spacecraft". Simple, answered the authors: the space travelers have to attempt communication with a more advanced civilization, screaming "help" in all possible language and form as they are pushed into space!

Pushing atoms

Consider an atomic beam of lithium, irradiated transversely by radiation at 670 nm. Calculate the recoil velocity of one atom that has absorbed a photon of light. Calculate the corresponding kinetic energy of that atom. The recoiling atom sees the 670 nm radiation Doppler shift by $\Delta\omega_D$. Calculate this Doppler shift and compare $\hbar\Delta\omega_D$ with the kinetic energy of the atom.

Solutions

Problem 1

10% of the power for 200 seconds corresponds to an energy applied to acceleration of 20 MJ $= mv^2/2$. The velocity is thus:

$$v = \sqrt{\frac{2 \times 20 \cdot 10^6}{1000}} = 200\text{m/s}. \tag{1}$$

In the case of the reflecting mirror, the total number of photons that have impinged on the mirror in 200 s is $200\text{MJ}/\hbar\omega = N$. The recoil momentum per photon is $(\hbar\omega/c)$. In the case of a mirror, there is a double recoil (the photon that got onto the mirror gives on recoil, another recoil as the mirror spits the photon back out). The change in momentum mv (m is the mass, v the velocity, and we start from a zero initial velocity) is equal to the change of momenta due to the photons.

$$mv = 2N\frac{\hbar\omega}{c} = \frac{200\text{MJ}}{\hbar\omega}\frac{2\hbar\omega}{c} = \frac{4 \cdot 10^8}{c} = \frac{4}{3}. \tag{2}$$

The velocity is thus:

$$v = \frac{4}{3}10^{-3}\text{m/s} \tag{3}$$

Problem 2

The atomic mass of lithium is 7. The frequency corresponding to the wavelength of 670 nm is $\omega = 2\pi c/\lambda = 2.8 \cdot 10^{15}$ s^{-1}. The recoil velocity of the atom is calculated from the equality of momenta: $Mv = \hbar\omega/c$:

$$v = \frac{\hbar\omega}{Mc} = \frac{1.0545 \cdot 10^{-34} \times 2.8 \cdot 10^{15} \times 6.022 \cdot 10^{23}}{7 \times 3 \cdot 10^8} = 0.000085\text{m/s}. \tag{4}$$

The kinetic energy of the atom is:

$$M\frac{v^2}{2} = \frac{\hbar\omega}{Mc^2}\frac{\hbar\omega}{2} = 4.2 \cdot 10^{-32}J. \tag{5}$$

The energy corresponding to the Doppler shift is:

$$\hbar\Delta\omega_D = \hbar\frac{v}{c}\omega = \frac{\hbar\omega}{Mc^2}\hbar\omega = 8.4 \cdot 10^{-32}\text{J}, \tag{6}$$

which is just twice the single recoil energy.

If the light pulse were to be sent back, the beam would recover a photon of a lower energy [by the amount given by Eq. (6)], while the atom would double its kinetic energy, and the energy would be conserved.