Nonlinear Optics — Homework 2

Due Monday, February 12, 2024

Polarization gate

Other than the polarization gate using a multiple order $\lambda/4$ wave plate plus a zero order $\lambda/4$ wave plate that we discussed in the class, another version will be a multiple order full (or half) λ wave plate plus a zero order $\lambda/4$ wave plate.

Start with an electric field of

$$E(t) = 2\mathcal{E}(t)\cos(\omega t) \tag{1}$$

$\mathcal{E}(t)$ is the envelope of the ultra short pulse. Assume that the envelope has a Gaussian shape, i.e.

$$\mathcal{E}(t) = E_0 e^{\frac{-t^2}{t_p^2}} \tag{2}$$

$\tau_p = 5$ fs is the pulse width.

The main goal is to have linear at the ends where the pulses do not overlap, and somewhere a quarter wave inside where the two pulses overlap. It can be half or full wave. In the half wave case, the electric field becomes:

$$E(t) = \sqrt{2}[E_0^-(t')\hat{i} + E_0^+(t')\hat{j}]\cos(\omega t')$$

$\hat{i}$ and $\hat{j}$ being the fast axes and slow axis of the wave plate. In the time coordinate t', the angle $\theta(t')$ of the polarization is:

$$\begin{aligned}\theta(t') &= \tan^{-1}[\frac{E_0^+(t')}{E_0^-(t')}] \\ &= \tan^{-1}(e^{-\frac{2\tau t'}{\tau_p^2}})\end{aligned}$$

The angle $\theta(t')$ is plotted as function of t' in Fig. 1.

A zero order $\lambda/4$ wave plate is placed at an angle θ_2 with respect to the multiple order wave plate.

After the $\lambda/4$ wave plate, the electric field becomes:

$$\begin{aligned}E_{x'}(t') &= \sqrt{2}\cos(\omega t)[E_0^-(t')\cos\theta_2 + E_0^+(t')\sin\theta_2] \\ E_{y'}(t') &= -\sqrt{2}\sin(\omega t)[E_0^+(t')\cos\theta_2 - E_0^-(t')\sin\theta_2]\end{aligned}$$

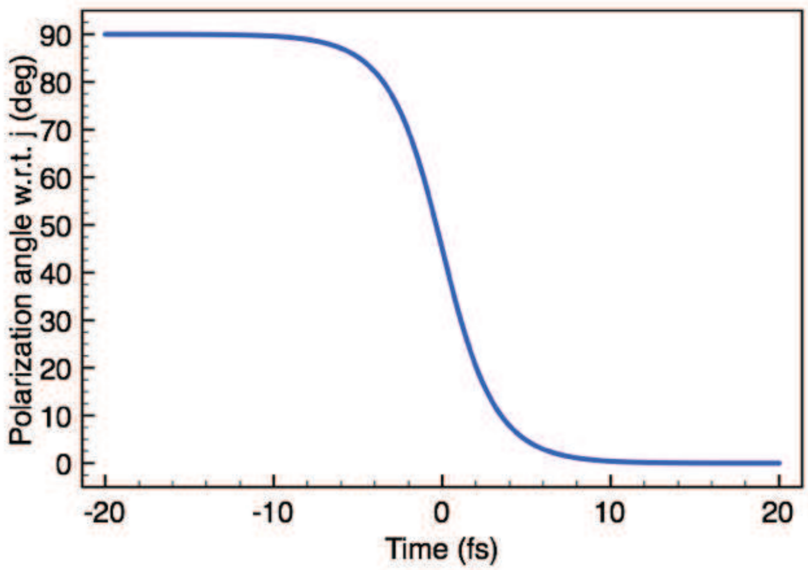

Figure 1: Polarization angle of the ultra-short pulse after a multiple full waveplate. After a multiple half wave the angle would change from + 45 to - 45°.

This is an elliptical polarized pulse with its major and minor axis on x' and y'. Its time dependent ellipticity can be described as:

$$\epsilon(t') = \left|\frac{E_0^-(t')\sin(\theta_2) - E_0^+(t')\cos(\theta_2)}{E_0^-(t')\cos(\theta_2) + E_0^+(t')\sin(\theta_2)}\right|$$

With $\theta_2 = 45^o$, the ellipticity is:

$$\begin{aligned}\epsilon(t') &= \left|\frac{E_0^-(t') - E_0^+(t')}{E_0^-(t') + E_0^+(t')}\right| \\ &= \left|\frac{e^{\frac{2\tau t'}{\tau_p^2}} - 1}{e^{\frac{2\tau t'}{\tau_p^2}} + 1}\right|\end{aligned}$$

Figure 2 shows the time dependent ellipticity (blue solid line) as compared to the input pulse intensity profile (black dashed line).

Note that the apparent discontinuity is due to defining the ellipticity as only a positive number.

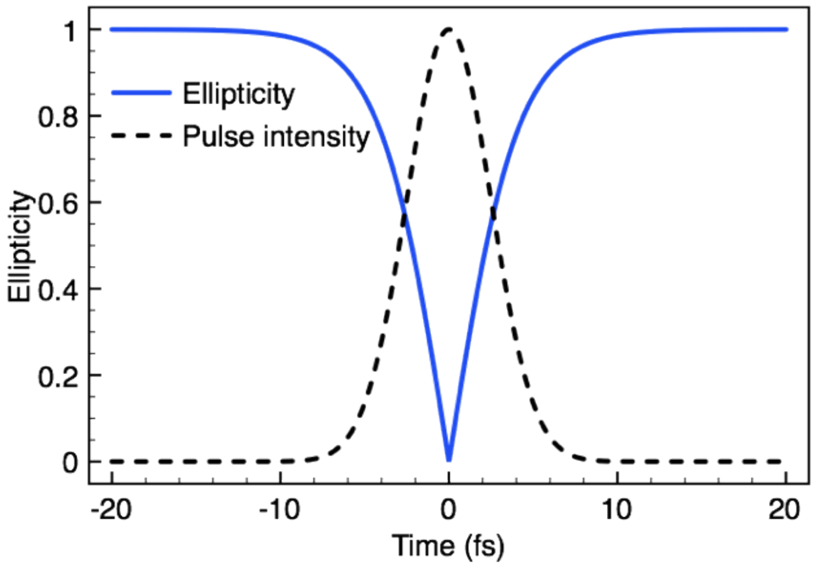

Figure 2: Time dependent ellipticity after the zero order $\lambda/4$ waveplate

Suppose the threshold ellipticity to create the 25th harmonics is 0.12, let us

calculate the polarization gate width.
Using the expression of the time dependent ellipticity, the gate width can be calculated by:

$$\begin{aligned}\tau_{\text{gate}} &= \frac{\tau_p^2}{\tau}\ln\left(\frac{1+\epsilon_{\text{th}}}{1-\epsilon_{\text{th}}}\right) \\ &= \frac{5^2}{6.2}\ln\left(\frac{1+0.12}{1-0.12}\right)\text{ fs} \\ &= 0.9724\text{ fs}\end{aligned}$$

which is less than half an optical cycle.