Laser Physics 464 — Homework 3

Due Wednesday, October 11, 2023

"Pot pourri" of parameters

You are given the absorption cross section of a medium (σ) and the density. Find all other parameters (spontaneous relaxation rate T_1, saturation energy density, saturation intensity, Rabi frequency coefficient, dipole moment, absorption coefficient, *etc* ... (in case I forgot some), At each step check the dimensions.
Numerical example: $\sigma = 5 \cdot 10^{-16}$ cm^2; $N = 5 \cdot 10^{18}$ cm^{-3}; $\lambda = 800$ nm.

Solution

Einstein coefficients and rate equations

There is some confusion in Verdeyen in transferring from Einstein coefficient equations to the rate equations. Einstein coefficient equations consider light interacting with a $\rho(\nu)$ density of photons with frequency between ν and $\nu + d\nu$ at the location of the two-level system. The population transfer between the two levels are related through:

$$\frac{dN_2}{dt} = -A_{21}N_2 - B_{21}N_2\rho(\nu) + B_{12}N_1\rho(\nu) = -\frac{dN_1}{dt}, \tag{1}$$

where A_{21} and B_{21} are the Einstein's coefficients for spontaneous and stimulated emission, $B_{21} = c^3/(n^3 8\pi h\nu^3)A_{21}$.

In the standard rate equation:

$$\frac{d\Delta N}{dt} = -\frac{I\Delta N}{W_s} - \frac{\Delta N - \Delta N_0}{T_1} \tag{2}$$

the radiation of intensity I applies to the full line of width $1/T_2$.

$$I = \rho(\nu) \times \frac{c}{T_2}$$

. Identifying Eqs. (1) and (2):

$$A_{21} = \frac{1}{2T_1} = \frac{n^3 8\pi h\nu^3}{c^3}\frac{c}{nT_2}\frac{1}{W_s} = \frac{8\pi n^2\nu^2\sigma}{T_2 c^2} \tag{3}$$

Energy relaxation time T_1

$$T_1 = \frac{T_2 c^2}{16\pi n^2 \nu^2 \sigma}$$

Saturation intensity

$$I_s = \frac{n}{2\kappa^2 T_1 T_2 \sqrt{\mu_0/\epsilon_0}} = \frac{\hbar\omega}{\sigma T_1}$$

Rabi frequency coefficient

$$\kappa = \frac{p}{\hbar} = \sqrt{\frac{n\sigma}{\hbar\omega T_2 \sqrt{\mu_0/\epsilon_0}}}$$

Dipole moment

$$p = \sqrt{\frac{\hbar n\sigma}{\omega T_2 \sqrt{\mu_0/\epsilon_0}}}$$

Absorption coefficient

$$\alpha = N\sigma$$