Optics 463 — Homework 4
due Wednesday, September 16

Ray tracing problem

Positive-negative lenses - solve with a ruler

Consider a positive lens of focal distance 8 cm, followed by a negative lens of 2 cm focal length. At which distance would you put the two lenses so that rays parallel to the optical axis emerge (after the two lenses) parallel to the axis?

Having chosen this distance, you send a beam parallel to the axis, 1 cm above the axis, towards the positive lens. At which distance from the axis (and which side) does the beam emerge from the two lens combination? Express the ratio of this distance to that of the initial distance in terms of the focal distances.

Consider now an incident ray on this same two-lens system making an angle of θ_i with the optical axis. What will be the angle θ_o of the emerging beam? Express this angle in terms of the focal distances.

The Drop

Considering Fig. 1, show that there is an angle of incidence α_i for which the angle β is maximum. β is the angle between the incident ray ant the ray exiting the sphere after one internal reflection. With the index of the drop equal to 1.33, find the value of the maximum angle β.

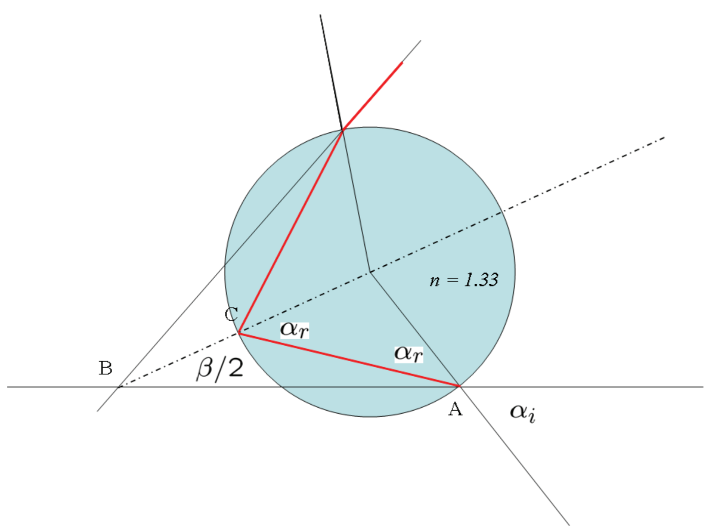
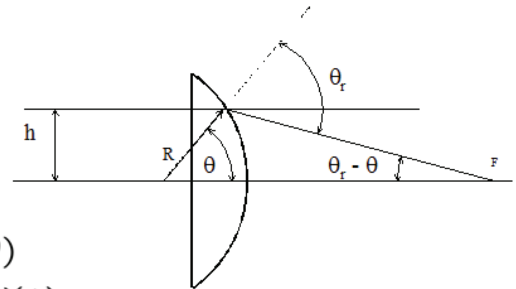


Figure 1: Drop geometry

Plano-convex versus convex plano

Demonstrate which orientation is the best. You can make a second order approximation to the paraxial approximation. The derivation for the plano-convex lens is given in Fig 2. To compare, make a similar derivation for the convex-plano [sketch 3.



$$h = R\sin\theta \;=\; f\tan(\theta_r - \theta) \approx f\sin(\theta_r - \theta)$$
$$\;=\; f\,(n\sin\theta\cos\theta - \cos\theta_r\sin\theta) \qquad (1)$$

$$f = \frac{R}{n\cos\theta - \sqrt{1 - n^2\sin^2\theta}}. \qquad (2)$$

To second order: $\cos\theta = 1 - \theta^2/2$; $n^2\sin^2\theta \approx n^2\theta^2$; and $\sqrt{1+x} \approx 1 + x/2$.
focal length:

$$f \;\approx\; \frac{R}{n(1-\frac{\theta^2}{2}) - (1 - \frac{1}{2}n^2\theta^2)}$$
$$\;=\; \frac{R}{n - 1 + n(n-1)\frac{\theta^2}{2}} \qquad (3)$$

Figure 2: Plano-convex

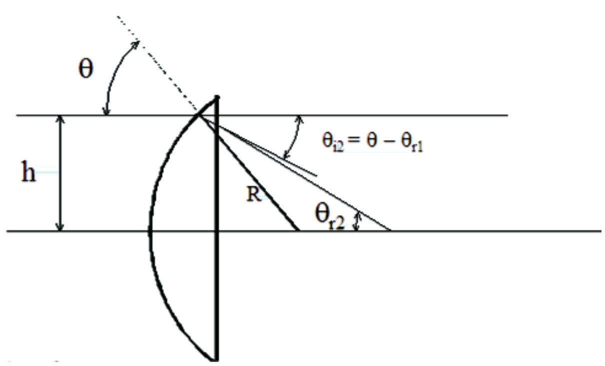


Figure 3: Convex-plano