Optics 463 — Homework 3
due Tuesday, September 25

Parabolic reflector

Demonstrate that all rays parallel to the axis (or orthogonal to the directrix) converge to the focus.

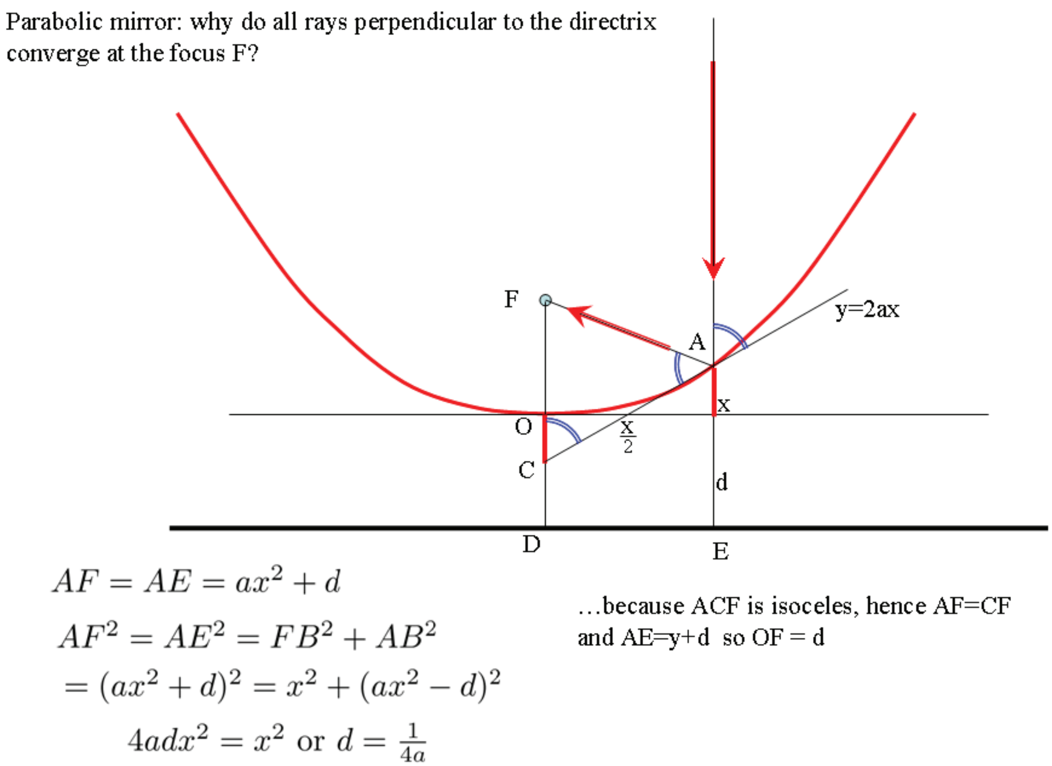


Figure 1: Solution

The Drop

Considering Fig. 1, show that there is an angle of incidence α_i for which the angle β is maximum. β is the angle between the incident ray ant the ray exiting the sphere after one internal reflection. With the index of the drop equal to 1.33, find the value of the maximum angle β.

From the triangle ABC we get:

$$\frac{\beta}{2} + (\alpha_i - \alpha_r) = \alpha_r. \tag{1}$$

which gives a relation for β versus the $alpha$'s which we differentiate to find the extremum:

$$\frac{d\beta}{d\alpha_i} = 2\frac{d\alpha_r}{d\alpha_i} - 1 = 0. \tag{2}$$

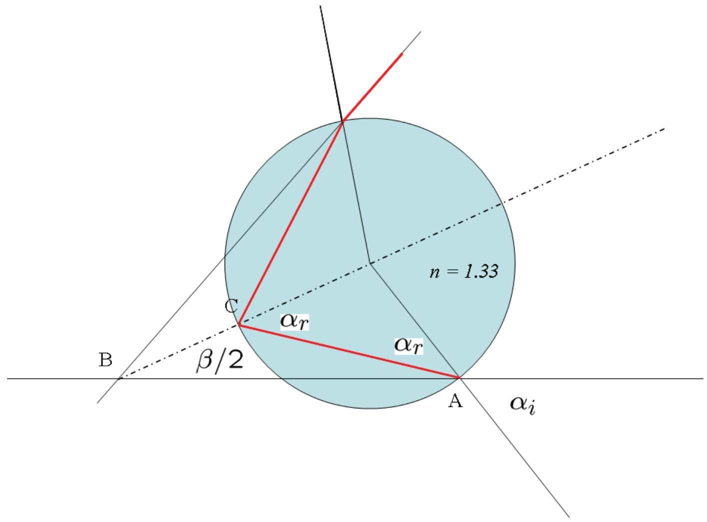


Figure 2: Drop geometry

The solution is:

$$\frac{d\alpha_r}{d\alpha_i} = \frac{1}{2} = \frac{cos\alpha_i}{n\cos\alpha_r}. \tag{3}$$

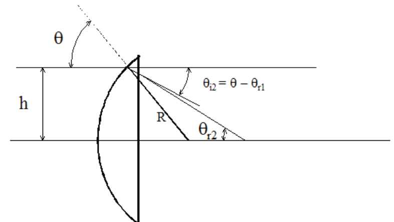
This reduces to the simple equation:

$$x^2 = \frac{4 - n^2}{3} = 0.7437$$

for $n = 1.33$, where $x = \sin\alpha_i$. That gives $\alpha_i = 59^o$.

Plano-convex versus convex plano

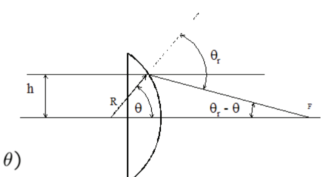
Demonstrate which orientation is the best. You can make a second order approximation to the paraxial approximation.



$$\begin{aligned} h = R\sin\theta &= f\tan\theta_{r,2} \approx f\sin\theta_{r,2} \\ &= fn\sin(\theta-\theta_{r,1}) = fn\left(\sin\theta\cos\theta_{r,1} - \sin\theta_{r,1}\cos\theta\right). \end{aligned}$$

$$\begin{aligned} f &= \frac{R}{\sqrt{n^2-\sin^2\theta}-\cos\theta} \\ &\approx \frac{R}{n(1-\frac{\theta^2}{2n^2})-1+\frac{\theta^2}{2}} \\ &= \frac{R}{n-1+\frac{\theta^2}{2}(1-\frac{1}{2n})}. \end{aligned}$$

Figure 3: Convex-plano



$$\begin{aligned} h = R\sin\theta &= f\tan(\theta_r-\theta) \approx f\sin(\theta_r-\theta) \\ &= f\,(n\sin\theta\cos\theta - \cos\theta_r\sin\theta) \qquad (1) \end{aligned}$$

$$f = \frac{R}{n\cos\theta - \sqrt{1-n^2\sin^2\theta}}. \qquad (2)$$

To second order: $\cos\theta = 1-\theta^2/2$; $n^2\sin^2\theta \approx n^2\theta^2$; and $\sqrt{1+x} \approx 1+x/2$.
focal length:

$$\begin{aligned} f &\approx \frac{R}{n(1-\frac{\theta^2}{2})-(1-\frac{1}{2}n^2\theta^2)} \\ &= \frac{R}{n-1+n(n-1)\frac{\theta^2}{2}} \qquad (3) \end{aligned}$$

Figure 4: Plano-convex