Optics 463 — Homework 6
due Wednesday, October 7, 2020

Derive the evolution equations of the q parameter

The Gaussian is defined as:

$$\tilde{\mathcal{E}}(x,y,z) = \frac{\mathcal{E}_0}{w(z)} e^{-i\Theta} \times e^{-ik_\ell (r^2)/2\tilde{q}}. \tag{1}$$

Substitute in Maxwell's propagation equation (paraxial approximation):

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - 2ik_\ell \frac{\partial}{\partial z} \right) \tilde{\mathcal{E}}(x,y,z) = 0, \tag{2}$$

to derive the z dependence of q, P and Θ.

Applying ABCD matrix to sphere

Consider a sphere having a radius R, an index n, and surrounded by air ($n_{air} = 1$), as in Fig. 1. Assume all paraxial rays.

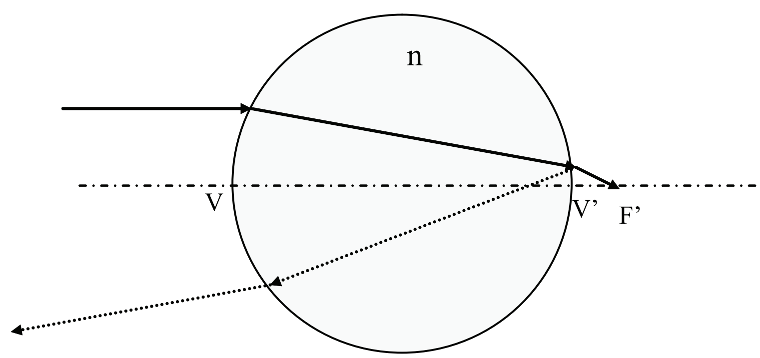

Figure 1:

a) Using optical matrices, derive an expression for the position of the image focal point (F') with respect to V'.

b) Give the definition of the "principle planes" in general and then set up matrices that will give the location of these planes for the above lens.

c) Find out the condition (i.e. the index) for which the reflected paraxial rays (shown by dashed lines) are parallel to the incident rays (retro-reflection). Show the ray paths for this case.

d) Is there a condition (i.e. a refractive index) for which the above sphere will act as a telescope (that is; an incident collimated laser beam will exit the sphere collimated)? Explain your result.

Hints for quick solutions to parts (c) and (d): Use the results from part (a) and symmetry arguments.