Optics 463 — Homework for Tuesday, October 9, 2018

Matrix 1

Consider a sphere having a radius R, an index n, and surrounded by air ($n_{air} = 1$), as in Fig. **??**. Assume all paraxial rays.

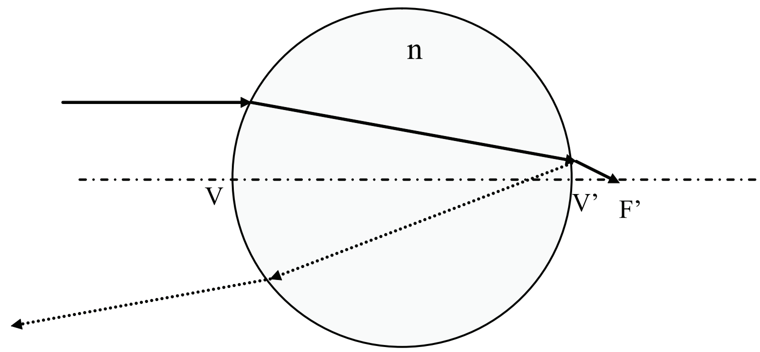

Figure 1:

a) Using optical matrices, derive an expression for the position of the image focal point (F') with respect to V'.

b) Give the definition of the "principle planes" in general and then set up matrices that will give the location of these planes for the above lens.

c) Find out the condition (i.e. the index) for which the reflected paraxial rays (shown by dashed lines) are parallel to the incident rays (retro-reflection). Show the ray paths for this case.

d) Is there a condition (i.e. a refractive index) for which the above sphere will act as a telescope (that is; an incident collimated laser beam will exit the sphere collimated)? Explain your result.

Hints for quick solutions to parts (c) and (d): Use the results from part (a) and symmetry arguments.

Solutions

(a) First transmission followed by a translation of 2R:

$$y\begin{pmatrix} \frac{2-n}{n} \\ \frac{(1-n)}{nR} \end{pmatrix} = y\begin{pmatrix} 1+\frac{2(1-n)}{n} \\ \frac{(1-n)}{nR} \end{pmatrix} = \begin{pmatrix} 1 & 2R \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1-n}{nR} & \frac{1}{n} \end{pmatrix}\begin{pmatrix} y \\ 0 \end{pmatrix} \tag{1}$$

The focus is at the second interface if $(2-n)/n = 0$ or $n = 2$. Next interface:

$$y\begin{pmatrix} \frac{2-n}{n} \\ \frac{2(1-n)}{nR} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{(n-1)}{R} & n \end{pmatrix}\begin{pmatrix} \frac{2-n}{n} \\ \frac{(1-n)}{nR} \end{pmatrix} y \tag{2}$$

Interesting case: $n = 2$: the slope in the sphere is $-y/2R$; at the second interface it simply doubles to $-y/r$, as it should be. Next we propagate to the focus, distance ℓ from the right interface:

$$y\begin{pmatrix} \frac{2-n}{n} + \frac{2\ell}{nR}(1-n) \\ \frac{2(1-n)}{nR} \end{pmatrix} = \begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{2-n}{n} \\ \frac{2(1-n)}{nR} \end{pmatrix} y \tag{3}$$

The condition to reach the focus being:

$$\frac{2-n}{n} + \frac{2\ell}{nR}(1-n) = 0$$

, the focus is at

$$\ell = -\frac{R(2-n)}{2(1-n)}$$

from the second surface.

(b) We start from the second interface, and propagate back a distance x:

$$\begin{pmatrix} y \\ whatever \end{pmatrix}\begin{pmatrix} 1 & -x \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{2-n}{n} \\ \frac{(1-n)}{nR} \end{pmatrix} y \tag{4}$$

x is then the solution of the equation:

$$1 = \frac{2-n}{n} - x\frac{2(1-n)}{nR}$$

(c) Retro-reflection: focus on axis at the exit interface of the sphere, which occurs for $n = 2$ (or $\ell = 0$ in (a).

(d) The solution is (realistic if you are a theoretician) $n = \infty$ will bring the ray through the center.