PHYC463: **Advanced Optics I**

INTRODUCTION

1 Light (photon) is a wave

In modern language we say that light consists actually of photons, just as matter is made of atoms. Our intuitive picture of atoms is that they can nicely be classified by their mass in a table — the Mendeleev table. Atoms themselves are boxes filled with electrons, protons and neutrons, and there is a mass associated with each component. Molecules and atoms are what we deal with on a daily base, but at a such a fine scale that it escapes our direct perception. Photons are as ubiquitous, but quite different from atoms and their constituents. Ubiquitous, because they are associated not only with visible light, but also with invisible radiation (infrared and ultraviolet), x-rays, gamma rays, radio waves, and even the radiation from our electrical network at 50 or 60 Hz. Quite different, because there is no mass associated to the photon. A wave is associated with the photon, which is an oscillation propagating at the speed of light.

What is a wave? There is always a pattern and motion associated to the wave; the ripples of a stone thrown in a pond or folds of a flag. One can imagine more and more of the examples that the word "wave" is applied to. If you take a picture from the ripples on the pond you realize that there are regular patterns that repeat in water, and you can possibly count the number of peaks on the water surface, that are separated by "wavelength". You can also only consider a fixed point on the surface, and monitor its motion as it goes up and down, or oscillates. It takes a "period" for each point on the pond to repeat its position. The pattern on the wave (for example the peaks) have a certain "speed" or "wave velocity", and the peaks that are created by the wave have an "amplitude". It is reasonable to conclude that stronger waves have bigger amplitudes, but there is more to the strength or energy of a wave.

When a wave goes through a medium it does not mean that the medium is necessarily moving with it. In the case of a flag waving in the wind, there is a wave that goes through the flag, but the fabric itself is not carried away. The wave propagate for huge distances, while each particle responsible for the wave motion stays at the same average position, just inducing the motion of the next particle. In most cases, the wave starts from a local oscillation [Fig 1 (a)], and propagates radially from there, like rings produced by a duck paddling on a pond [Fig. 1 (b)]. In the case of light, it is the electric field produced by a charge oscillating up and down that starts off the wave. This is called *dipole* emission.

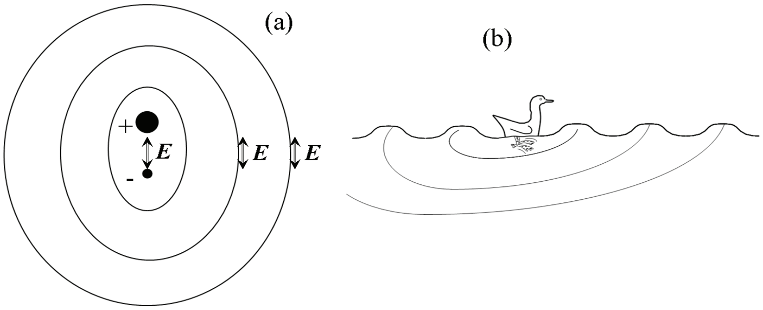

Figure 1: (a) an oscillating electric field is created by a pair of charges with a periodically varying distance (oscillating dipole). This periodically varying field creates an electromagnetic wave that propagates at the speed of light in vacuum, just as a wave is created in (b) by a duck paddling in a pond.

The electric field produces a magnetic field that produces an electric field, and that is essentially how light waves propagate. The velocity of a wave is a property of the medium in which the wave propagates. Sound waves propagate at 343 m/s (1,125 ft/s) in dry air at room temperature, and faster in denser media. It is the opposite for the light waves which usually travel faster in air or vacuum. There are different types of waves. Mechanical waves like spring oscillations and sound waves are due to mechanical motion of particles. The oscillation of these waves is along the propagation direction. Light waves, however, are electromagnetic waves, which originate from the oscillation of charges (electrons for example). This was the first dilemma in early attempts to interpret light waves: what is moving?

Our intuition is shaped by the observation on water waves in a pond, an oscillating spring, or the swing of a pendulum. These are all mechanical waves. Like sound waves, they require a medium; they need matter to exist. Hence was born the notion of the "ether", a (fictitious?) medium to support the propagation of light waves. Today the"ether" has simply given way to vacuum, but it does not mean that the understanding of the nature of light has become simpler.

Quantum mechanics tells us that the amplitude of the positive-negative charge oscillation is restricted to discrete values. Consequently, the emitted oscillation takes also discrete values, to which is associated an energy: the photon energy $h\nu$, where ν the frequency of the oscillation, and h is called the "Planck constant" [see Eq. (1)]. It is as if the duckling in Fig 1 (a) had discrete gears to activate his webbed paws. What is more puzzling is that the "neutral" gear is missing. The minimum energy state of the quantum harmonic oscillator is not zero, but $(1/2)h\nu$. This is often referred to as *vacuum fluctuation* or the zero point energy. The absence of vacuum (the ether concept) has been replaced by an absence of zero energy. Since, according to Einstein, there is an equivalence of matter and energy, the two concepts are not so far apart.

2 Photons

2.1 Photon energy

Quantum mechanics tells us that a photon has dual characteristic, it acts both as a wave (section 1) and as a particle. In a way photon is a wave that can be counted. This might be a bit hard to digest, since our common sense is restricted to our daily experience with objects that are not so delicate. What do we mean by acting like a particle? They can be counted. A photon is like a "currency", and the light that we experience is like a sum of money, we never notice that tiny penny.

Let us take a closer look and see why we generally ignore single photons. A typical red laser pointer has an output power of 3mW (3 mJ/s), which consists of individual photons having an energy of the order of $3 \cdot 10^{-19}$ J. It means that every second there are 10000 million million photons shooting out of a pointer. If we even associate a penny to each photon, in a second we get a sum of money that is more than the wealth of a country.

Just as not all currencies have the same value, the photons have different energies. Here we need to use the wave aspect of the photon. The faster a wave oscillates, the more energy it possesses.

The longest (slower) electromagnetic wave that we encounter in our daily life is created by the 50 Hz electrical network covering the globe. As a result the earth radiates, making one oscillation over a distance of 6000 km. Radio waves are long too: it takes 3 meter (3.3 yard) for a short wave (FM radio) to make an oscillation. For a long wave (AM) it takes about 300 meter (330 yard) to complete one.

The visible light that we are used to also oscillates, but much faster. The green visible light consists of photons of 500 nanometer wavelength; meaning that over a thickness of a sheet of paper (which is 0.1 mm or 0.004 of an inch) it makes 200 oscillations. An X-ray with wavelength of about one nanometer, oscillates 100000 times over the same length. It appears thus that the following connection exists: photons that oscillate faster have a shorter wavelength, and more energy. Or in the simplified language of mathematics

$$E = h\nu = \frac{hc}{\lambda}, \tag{1}$$

where "E" stands for energy, "h" is the physical Plank's constant, "c" is the speed of light , "ν" is the number of oscillation of a photon in a second, and "λ" is the wavelength, or the length in which a single oscillation takes place. For the photon associated with visible radiation, the elementary photon energy is too small to be using the traditional energy unit of Joule. Instead, the energy unit used by physicists is the electron-volt (eV). One eV ($1.602 \cdot 10^{-19}$ J) is the energy acquired by an electron that is accelerated under the potential difference of one volt. Infrared radiation at a wavelength of 1.24 μm has exactly the energy of 1 eV. As shown in Fig. 2, our earth, due to the electric power network, radiates photons of $2.067 \cdot 10^{-13}$ eV energy.

2.2 Energy and size

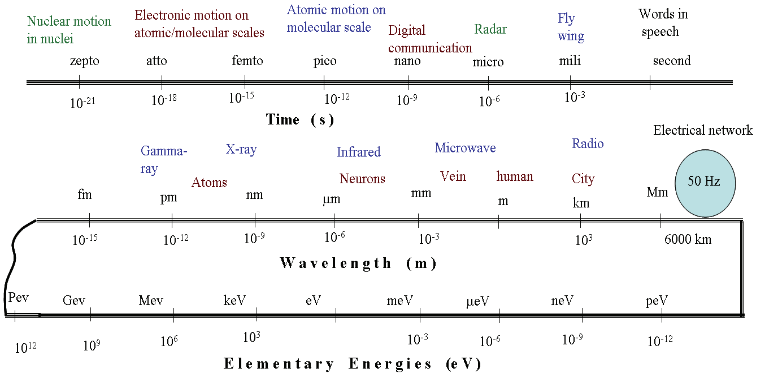

Figure 2: (a) The different objects that radiate electromagnetic waves; wavelength and elementary energy associated with them.

Could "spiderman"' really have the strength of a spider, scaled up to his size? Is a cat that is 100 times more massive than a mouse 100 times stronger? In biology things won't scale linearly. Body mass increases linearly with volume in three dimensions while muscle strength in arms and legs are proportional to cross sections, and therefore they increase only in two dimensions. If a human is a million times more massive than an ant, he is only 10000 times stronger. In a way smaller animals are stronger relative to their masses. Physics scales in a simpler way than biology. In a musical instrument higher frequencies are generated by shorter strings, thus have more energy. Some physicist like to draw a box around the object that they study, and they know as the box gets smaller they are dealing with higher and higher energies. The speed and energy of the electrons oscillating in an atom are much bigger than the ones traveling in a long wire loop.

Using our wave picture and the equation of photon energy (1) we can look more closely at the size-energy relation. Consider fitting one full wave in two different size boxes. The wave that fits in the smaller box [Fig. 3 (b)] has a shorter wavelength than the one in the bigger box [Fig. 3 (a)]. Using the photon energy equation [Eq. (1)], the wave with a smaller wavelength has higher energy. It seems the more confined the wave, the stronger its elementary energy. This seems like an oppression force! Quantum mechanics tells us that the electrons around an atom are confined to well defined shells or electron levels like Russian nesting dolls [Fig. 3 (c) and (d)]. The electrons in bigger shells have less energy and are loosely bound, which is why in most ionization processes the chance of knocking off an electron from an outer shell is the highest. This order is not as rigid as the order of taking out the Russian dolls: when dealing with higher energies photons, it is possible to scoop up the electron from an internal shell, leaving the external ones in place (not something you could to with the Russian dolls).

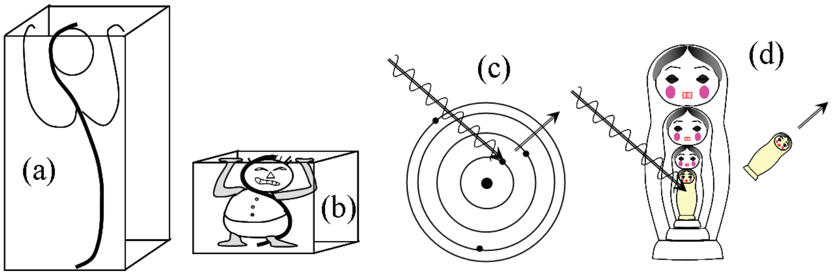

Figure 3: Particle and wave in a box: The longer wavelength fits in the larger box (a). A shorter wavelength fits in the smaller box (b), corresponding also to a larger particle energy. Electrons orbiting around the nucleus (c) are analog to nested Russian dolls (d)]. An photon of sufficient energy can knock off the electron of the outer shelve, as one can easily remove the outer layer of the nested dolls. More problematic is the removal of an inner shelve electron. While it would be an unresolvable "chinese puzzle" to remove the inner doll, the possibility to eject an inner shelve electron exists with high energy photons.

Looking at the waves in boxes, we were only concentrated on the concepts of size, energy and wavelength. There are other parallel manifestations of waves, such as in time and frequency. This means that in smaller boxes we are dealing with shorter time scales and faster frequencies. This is a very important fact for observing phenomena in nature: we need the proper time scale to watch an event. The speed that we take our data, or snapshots of an event, tells us what we can record and what we would miss. Imagine that we could only understand one word per minute. It would then be very difficult to get the meaning of sentences in a normal conversation. In Cinematography 24 frames per second is sufficient to capture most daily events, but if we want to see faster events like a bullet passing through a target, we need to take more frames per second and then play it at a normal 24 frame per second rate. In Fig. 2 the time scale of some events are shown. From this diagram one can see that in order to observe electron motion in atoms and molecules one should have a camera that takes picture every femtosecond. It is hard to imagine how fast a femtosecond is. If one could read at the rate of one word per femtosecond the first Harry Potter book with 2.7 million words, the whole book would be finished in a few nanosecond. Or, in other words, you could read Harry Potter a thousand million times in just one second. An attosecond is to the second, as a second is to the age of the Universe ($0.4 \cdot 10^{18}$ s). Figure 2 illustrates the various sources of radiation that affect our lives, and how they are associated with energy, size and time.

2.3 Mass-less but not momentum-less

The energy relation (1) can be written in different forms:

$$\frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} = p \tag{2}$$

which defines the momentum of a photon (energy/velocity = momentum).

The photon is a particle that propagates at the speed of light. We have to use the special relativity equation to describe a particle propagating at relativistic velocities, which define momentum, particle wavelength and velocity as:

$$p = \frac{\sqrt{E^2 - m_0^2 c^4}}{c} \tag{3}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - m_0^2 c^4}} \tag{4}$$

$$v = \frac{pc^2}{E} = c\sqrt{1 - \frac{m_0^2 c^4}{E^2}}. \tag{5}$$

E is the sum of the rest energy $m_0 c^2$ and the kinetic energy, which is now defined as:

$$E_k = m_0 c^2(\gamma - 1) = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \tag{6}$$

There are two limits:

- *classical mechanics* $\gamma \rightarrow 1$
- *photon* $\gamma \rightarrow \infty$ which is only possible if $m_0 = 0$

Inertia

Light has some inertia, since it has some momentum $mv \rightarrow \hbar\omega/c$, which is not infinite. Inertia is still a valid concept with the photon. As opposed to mechanical properties, one can say that the "friction" associated with propagation of the photon is nearly zero. A good example is that of the laser gyro. As the mechanical gyroscope, the angular momentum is conserved and is zero for two counter-circulating pulses. Orders of magnitude: 1 g at 1 meter at $\Omega = 1$ turn/day $= 1$ μ radian/s: the angular momentum is:

$$mr^2\Omega = 10^{-9}\text{J.s} \tag{7}$$

and the kinetic energy is:

$$mr^2\Omega^2 = 10^{-15}\text{J} \tag{8}$$

For a pulse of 1 Joule circulating at the speed of light on a 1 m radius: the number of photons is 1 J/0.6 eV = 10^{19} photons. The angular momentum is:

$$10^{19} \times \frac{\hbar\omega}{c} \times 1\text{m} = 10^{19} \times 3 \cdot 10^{-28} = 10^{-9}\text{J.s}. \tag{9}$$

Instead of 1 Joule — let us consider 1 kW of continuous power circulating in a ring cavity. The same calculation as above leads to a kinetic energy of only 1 μJ! The energy is in fact huge, for such a small momentum. Conservation of angular momentum – if there is no friction to the shaft, or — in the case of optics — with the cavity or the frame.

2.4 Momentum and energy - how small, or how big?

The perception of what is an "intense" light pulse varies over a immense range. For the medical user, a beam that burns the skin can be qualified as intense, and this requires no more than 10 or 100 W/cm^2. A chemist will consider high the intensity that will break a chemical bond, which can be in the range of MW/cm^2 to GW/cm2. Most power hungry are the physicists. At the lower end of the scale, the atom-physicist will only get excited if the field of the light [1] reaches a sufficient value in one oscillation to extract an orbiting electron from the atom [Fig. 4 (a)], and pull it back at the next half cycle of the field [Fig. 4 (b)]. The disturbed electron comes back to the atom with a vengeance, producing burst of x-rays of incredible short duration, in the attosecond range [Fig. 4 (c)]. One attosecond, 1000 times

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[1] Remembering that light is an electric field that oscillates in time.

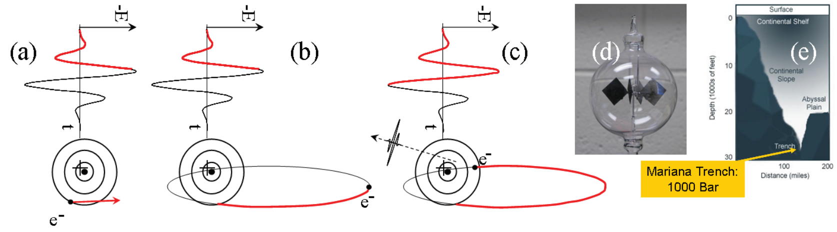

Figure 4: The electric field of a pulse, near the peak of a cycle, is sufficiently high to eject an electron from its orbit (a). The electron is accelerated away from the atom, then decelerated to a halt t the next half cycle, as the light electric field reverses sign (b). During the next quarter cycle of the light pulse, the electron is accelerated back towards the atom. Numerous complex physical phenomena can occur during that re-collision, one of them being the emission of an extremely short (typically 100 as) burst of x-ray radiation. (d) The pressure of sunlight or a He-Ne laser is not sufficient to make the "light mill" (or Crookes Radiometer") spin. However, the radiation pressure at intensities used to create attosecond pulses is 10,000 times larger than that at the bottom of the ocean (e).

shorter than the femtosecond, is in the same ratio to the second as the age of the universe is to the second (10^{18}). The intensities to reach electron extraction — return to the atom — are now in the terawatt (10^{12}) to petawatt (10^{15} W/cm^2 range. To move away from the abstraction of these numbers, let us try to get a feeling of what these intensity mean by considering the pressure of light. The photon has no mass, but it acts like a solid particle, having a momentum $h\nu/c$. When bouncing off a surface such as a mirror, it induces a recoil momentum $Mv = 2h\nu/c$, where v is the velocity given the mirror of mass M. A beam with a flux of N photons/(cm^2 s) has an intensity I (in Watts/cm^2), exerts a pressure of $N2h\nu/c = 2I/c$ on the mirror. Sunlight has an intensity of 0.1 W/cm^2. The pressure that sunlight exerts on the 1 cm^2 area of the reflecting "light mill" shown in Fig. 4 (d) is equivalent to on millionth of a 1 mg flee distributed over a cm^2 area! Definitely not sufficient to make a light mill spin[2]. At the intensity level of 10^{16} W/cm^2 used in attosecond pulse generation, the radiation pressure is 1,000 × higher than the pressure at the deepest point of the ocean [Fig. 4 (e)].

As impressive as these intensities may seem, there is another breed of physicists that look upon them with disdain. At sufficiently high intensities, electrons can be accelerated during a half optical cycle to relativistic velocities, i.e. velocities close to the speed of light. As one tries to accelerate more an electron by increasing the light field, the mass of the electron increases (to reach infinity at the speed of light). For the relativist plasma physicist, high intensity means above 10^{18} W/cm^2, for one micron radiation. That intensity corresponds to an optical field strength the accelerates the electron to a speed at which the electron mass has increased by 50%.

There is a group of of physicists even more greedy for power: the particle physicist. For him, an intense laser field E is such that, for instance, an electron would be accelerated over a wavelength λ_c, to an energy $eE\lambda_c \geq 2mc^2$ where m is the mass of the electron (or another particle). mc^2 is, according to Einstein, the energy equivalent to a particle of mass m at rest. λ_c is the wavelength associated with the accelerated electron, or "Compton wavelength".

A back of the envelope calculation shows that the corresponding laser intensity is above 10^{29} W/cm^2. The concentration of energy is such that the creation of matter — a pair of electron-positrons — will emerge from vacuum. A plethora of other effects are expected even below such intensity levels, such as changes in index of refraction.

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[2] One may wonder what makes a light mill spin? Each of the 4 planes of the light mill has a black face and a mirror face. The radiation pressure acts on the reflecting surface. One may notice that the irradiated mill turns as if the light was pushing the *black* surface. It is because the black surface absorbs the photon, heats the surface, which partially vaporizes. It is the recoil from the molecules escaping the black surface that makes the light mill spin.

One question to answer however is whether such intensities belong to science fiction, or real research. Numerous countries have national facilities in the PW (10^{15} W) range. The peak power of the next generation laser that is contemplated is above the "exawatt" (or 10^{18} W) range. Such a laser is no longer at the scale of a National laboratory, but requires resources at the scale of a continent. The European community has started such a project: the "Extreme Light Infrastructure" or ELI. Extreme is even an understatement, when trying to describe the highest intensities contemplated in this project. Instead of one facility, politics lead to the project being divide in three: in the Czech Republic, Hungary and Romania. The Czech Republic ELI focuses on short pulse secondary sources of particles. Hungary is concentrating on ultrashort pulses of high repetition rates. Roumania is supposed to focus on laser based nuclear physics.

Fermat principle

The light goes in a straight light because if follows the curve of shortest path. Thus straight line only in an homogeneous medium. This applies as well to media that are not homogeneous, and two dimensional surfaces (whispering gallery modes). Other example: Radio waves of low enough frequency trapped below the ionosphere.

Waves

Spherical waves in homogeneous media. Normals are the rays. Simple mechanical model: restoring force proportional to displacement of next neighbor:

$$C(x_n - x_{n-1}) + C(x_n - x_{n+1}) + M\frac{d^2x_n}{dt^2} = 0. \tag{10}$$

The first two terms are proportional to the left and right derivatives:

$$\begin{aligned} C(x_n - x_{n-1}) &= C\Delta z \left.\frac{dx}{dz}\right|_{-} \\ C(x_n - x_{n+1}) &= -C\Delta z \left.\frac{dx}{dz}\right|_{+}, \end{aligned} \tag{11}$$

Therefore, Eq. (10) takes the form:

$$\frac{\partial^2 x}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 x}{\partial t^2} = \left[\frac{\partial}{\partial z} - \frac{1}{v}\frac{\partial}{\partial t}\right]\left[\frac{\partial}{\partial z} + \frac{1}{v}\frac{\partial}{\partial t}\right] x = 0, \tag{12}$$

which has as solution forward and backward propagating waves.