Polarization gate

Other than the polarization gate using a multiple order $\lambda/4$ wave plate plus a zero order $\lambda/4$ wave plate another version will be a multiple order full λ wave plate plus a zero order $\lambda/4$ wave plate.

Start with an electric field of

$$E(t) = 2\mathcal{E}(t)\cos(\omega t) \tag{1}$$

$\mathcal{E}(t)$ is the envelope of the ultra shot pulse. Assume that the envelope has a Gaussian shape, i.e.

$$\mathcal{E}(t) = E_0 e^{\frac{-t^2}{t_p^2}} \tag{2}$$

$\tau_p = 5$ fs is the pulse width.

The electric field is first incident on a multiple full wave plate with its polarization axis at 45^o with respect to the fast axis of the wave plate. **The wave plate introduces a group delay of 6.2 fs between its e and o component at the central wavelength of the input pulse.**

A full wave plate give a phase retardation of 2π. So after the wave plate, the electric field becomes:

$$E(t) = \sqrt{2}[E_0^-(t')\hat{i} + E_0^+(t')\hat{j}]\cos(\omega t')$$

$\hat{i}$ and $\hat{j}$ being the F.A and S.A of the wave plate.

This is a linear polarized pulse with time dependent polarization angle θ w.r.t axis $\hat{j}$ (i.e. different portion of the pulse in time are polarized at different angle):

$$\begin{aligned} \theta(t') &= \tan^{-1}[\frac{E_0^+(t')}{E_0^-(t')}] \\ &= \tan^{-1}(e^{-\frac{2\tau t'}{\tau_p^2}}) \end{aligned}$$

The polarization angle versus time is shown in Fig. 1.

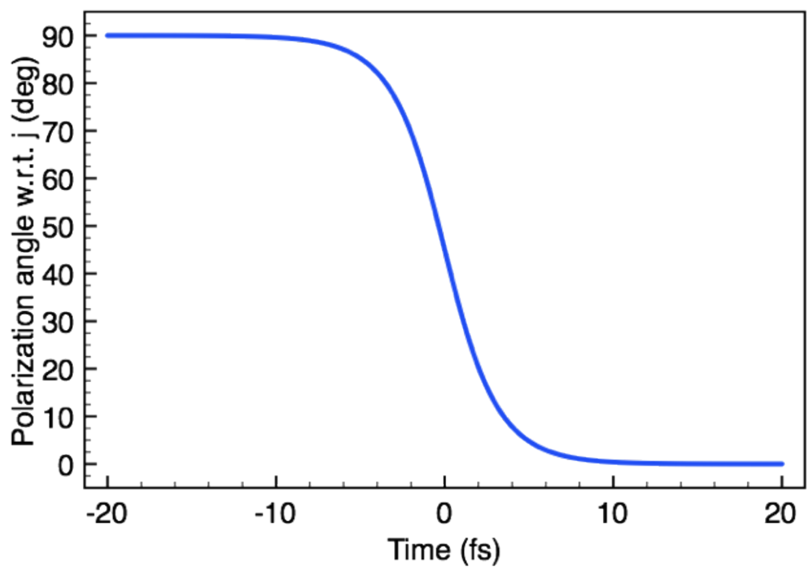

Figure 1: Polarization angle of the ultra-shot pulse after a multiple full waveplate

A zero order $\lambda/4$ wave plate is placed at an angle θ_2 with respect to the full wave plate. Describe the electric field and its time dependent ellipticity after the $\lambda/4$ wave plate. Plot the ellipticity as function of time at $\theta_2 = 45^o$.

Project the electric field on to the F.A. and S.A (x' and y'))of the zero-order $\lambda/4$ wave plate. After the $\lambda/4$ wave plate, the electric filed becomes:

$$E_{x'}(t') = \sqrt{2}\cos(\omega t)[E_0^-(t')\cos\theta_2 + E_0^+(t')\sin\theta_2]$$
$$E_{y'}(t') = -\sqrt{2}\sin(\omega t)[E_0^+(t')\cos\theta_2 - E_0^-(t')\sin\theta_2]$$

This is a elliptical polarized pulse with its major and minor axis on x' and y'. Its time dependent ellipticity can be described as:

$$\epsilon(t') = |\frac{E_0^-(t')\sin(\theta_2) - E_0^+(t')\cos(\theta_2)}{E_0^-(t')\cos(\theta_2) + E_0^+(t')\sin(\theta_2)}|$$

With $\theta_2 = 45^o$, the ellipticity is:

$$\epsilon(t') = |\frac{E_0^-(t') - E_0^+(t')}{E_0^-(t') + E_0^+(t')}| = |\frac{e^{\frac{2\tau t'}{\tau_p^2}} - 1}{e^{\frac{2\tau t'}{\tau_p^2}} + 1}|$$

See figure 2 for the time dependent ellipticity (blue solid line) as compared to the input pulse intensity profile (black dashed line).

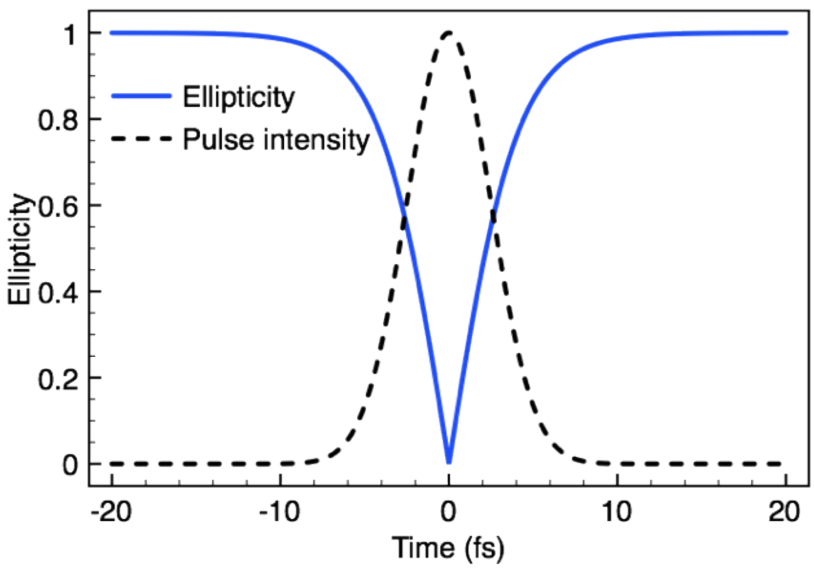

Figure 2: Time dependent ellipticity after the zero order $\lambda/4$ waveplate

Supposing the threshold ellipticity to create the 25th harmonics is 0.12, what is the polarization gate width?
Using the expression of the time dependent ellipticity in in the previous part, the gate width can be calculated by:

$$\begin{aligned}\tau_{\text{gate}} &= \frac{\tau_p^2}{\tau}\ln\left(\frac{1+\epsilon_{\text{th}}}{1-\epsilon_{\text{th}}}\right)\\ &= \frac{5^2}{6.2}\ln\left(\frac{1+0.12}{1-0.12}\right)\text{ fs}\\ &= 0.9724\text{ fs}\end{aligned}$$

which is less than half an optical cycle.