Optics 463, final Project
due on or before Monday, December 7, 2020

For details, see ppt file "SPACE-TIME ANALOGY" under **VII Space-time analogy**

1 Gaussian approximation, space cavity

1.1 Stable cavity, no time dependence

Find the q parameter, the beam size and the radius of curvature at the starting point marked as K. To make life easier, use fixed values in cm (for L, M, and d), but keep f (lens K) as a parameter.

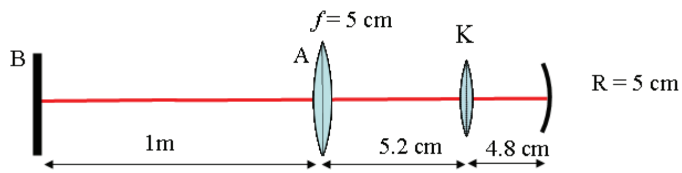


Figure 1: Laser cavity.

The ABCD matrix of this cavity is:

$$\begin{pmatrix} 2.44 & 0.288 \\ -7.2 - \frac{2.44}{f} & -0.44 - \frac{0.288}{f} \end{pmatrix} \tag{1}$$

This is the product of the matrix starting at K (no lens) by the lens matrix:

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 2.44 & 0.288 \\ -7.2 & -0.44 \end{pmatrix} \tag{2}$$

Choose f - 2.88 cm (lens at K). The pulse (beam) circulating in the cavity is a Gaussian

$$I_0 e^{-2t^2/\tau_g^2} e^{-2r^2/w^2}$$

with (initially) $I_0 = 2.43$ GW/cm^2. Calculate the pulse energy. The wavelength is 1 μm.

1. The initial Gaussian parameters of this cavity are given in slides 23-24 of "SPACE-TIME PROJECT" under VII. Space-time analogy.

2. Make a numerical program to apply this cavity to the q parameter.

3. The fixed parameter is the pulse energy (calculated from the chosen lensing f=2.88 cm, which, with a beam size of w =54 μm, corresponds to an intensity of 2.43 GW/cm^2, which with the beam size of w =54 μm corresponds to a power of 71 kW, which for a pulse duration of 100 fs gives a pulse energy of 7.1 nJ.

4. After each cycle, applying the ABCD matrix to the 1/q parameter of the previous cycle, the new 1/q gives a new beam size w, which given the fixed pusle energy, implies a changed intensity I_0, which means that you then have to re-calculate the lens f for the ABCD matrix of the next cycle.

5. Stability check: start with a Gaussian beam that is not the mode of the cavity. Let it propagate in the cavity, and see if the parameters converge towards the Gaussian solution.

6. If the result of the previous calculation does not stabilize to a steady state value, repeat the calculation after adding to the ABCD matrix the matrix —unity—$\times(i\epsilon)$ where $\epsilon \ll 1$ (for instance 0.05).

7. Kerr effect simulation: The lens K represents Kerr lensing in the cavity. Find the value of f that would de-stabilize the cavity.

2 Gaussian approximation, time cavity

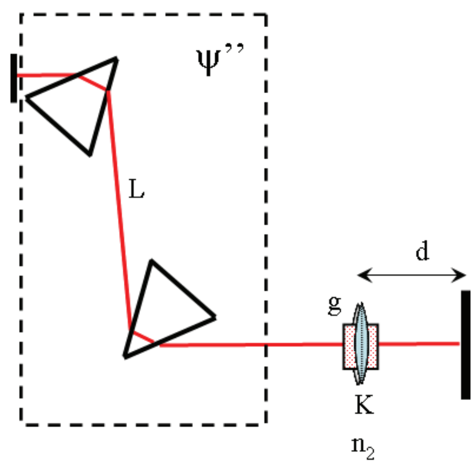


Figure 2: Cavity — time parameters

In the coarsest approximation, the cavity consists in a pair of prisms, and a Ti:sapphire crystal. We assume that gain and losses are linear and just balancing each other. We assume also that a Gaussian pulse circulates in the linear cavity sketched in Fig. 2, starting from the mirror on the left, propagating through a dispersive element consisting of two prisms of fused silica, then in the Ti:sapphire crystal, then returning to the Ti:sapphire crystal and the dispersive element. Find the (time) cavity round-trip matrix. Given the initial Kerr lensing of $\ddot{\Phi} = 0.64 \cdot 10^2$ ps^2, calculate the dispersion ψ'' consistent with the pulse duration of 100 fs as solution.

The parameters of the "time lens", is produced by the Kerr effect in the crystal at K.

$n_2 = 10.5 \cdot 10^{-16}$ cm^2/W
index of refraction $n = 1$
Length of crystal: 1 cm
Pulse duration $\tau_{g0} = 100$ fs

The beam cross section has been calculated in Section 1.1. The pulse energy is fixed.

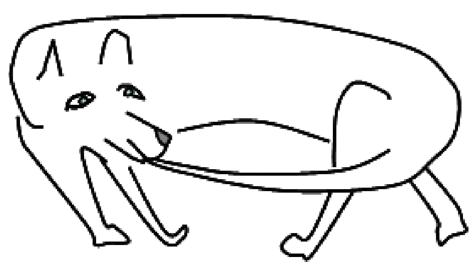
2.1 Cavity round-trips towards the steady state

Write the complex p parameter for an unchirped Gaussian pulse of duration τ_{g0}. Write a program to apply the cavity matrix to the p parameter repeatedly, and plot the pulse duration and chirp versus round-trip index.

2.2 Damped cavity

If the result of the previous calculation does not stabilize to a steady state value, repeat the calculation after adding to the ABCD matrix the matrix —unity—$\times(i\epsilon)$ where $\epsilon \ll 1$ (for instance 0.05).

3 The dog that bites its tail



The time and space cavities are not independent, but linked by the nonlinear lens K (for space) and the time lensing K (for time. Rather than solving independently for time and space, you need to cycle the cavity by successive space matrix (to determine the beam width w in the crystal) and time matrix (to determine the pulse duration, hence the intensity in the crystal).

There are two possibilities:

(a) The parameters converge towards a stable solution, giving beam waist, pulse duration, pulse energy, pulse chirp.

(b) The program blows up. A solution that would stabilize the program is a saturable nonlinear index:

$$n = n_0 + n_2 I - n_4 I^2, \tag{3}$$

where n_4 is a parameter to be determined to stabilize the solution.