Nonlinear non-resonant elements

(a) Self-phase modulation

Some elements impress a nonlinear phase upon the propagating pulse. This phase is the result of a nonlinear process of third-order and characterized by a nonlinear polarizability $\chi^{(3)}$. In the limit of a fast nonlinearity the response is instantaneous and is usually described by an intensity dependent refractive index. Acting only on the phase, such an element leaves the pulse envelope, $\mathcal{E}_0(t)$, unchanged. For interaction lengths much shorter than the dispersion length L_D, and for an instantaneous nonlinearity, the wave equation is:

$$\frac{\partial}{\partial z}\tilde{\mathcal{E}}(z,t) = -i\frac{3\omega_\ell^2\chi^{(3)}}{8c^2k_\ell}|\tilde{\mathcal{E}}^2|\tilde{\mathcal{E}} = -i\frac{n_2k_\ell}{n_0}|\tilde{\mathcal{E}}|^2\tilde{\mathcal{E}} \tag{1}$$

after applying the SVEA to the polarization term. For real $\chi^{(3)}$, substituting $\tilde{\mathcal{E}} = \mathcal{E}\exp(i\varphi)$ into Eq. (1) and separating the real and imaginary parts result in an equation for the pulse envelope

$$\frac{\partial}{\partial z}\mathcal{E} = 0 \tag{2}$$

and for the pulse phase

$$\frac{\partial\varphi}{\partial z} = -\frac{n_2k_\ell}{n_0}|\mathcal{E}|^2. \tag{3}$$

Obviously the pulse amplitude $\mathcal{E}$ is constant in the coordinate system traveling with the group velocity, that is, the pulse envelope remains unchanged, $\mathcal{E}(t,z) = \mathcal{E}(t,0) = \mathcal{E}_0(t)$. Taking this into account, we can integrate Eq. (3) to obtain for the phase

$$\boxed{\varphi(t,z) = \varphi_0(t) - \frac{k_\ell n_2}{n_0}z\mathcal{E}_0^2(t)} \tag{4}$$

which results in a phase modulation given by

$$\frac{\partial\varphi}{\partial t} = \frac{d\varphi_0}{dt} - \frac{n_2k_\ell}{n_0}z\frac{d}{dt}\mathcal{E}_0^2(t). \tag{5}$$

From Eq. (4)

$$\varphi(t,z) = \varphi_0(t) - \frac{k_\ell n_2}{n_0}z\mathcal{E}_0^2(t) = \varphi_0(t) - \frac{k_\ell\bar{n}_2}{n_0}zI_0^2(t). \tag{6}$$

If the actual profile of the incident beam is taken into account the index change becomes a function of the transverse coordinate, which leads to self-lensing effects. The general mechanism is described in Chapter 3; the effect of such an element in a fs laser is discussed in the next section.

(b) Polarization coupling and rotation

Nonlinear effects can also act on the polarization state of the laser pulse. This effect is used in some lasers (for instance in fiber lasers [1]) to produce mode-locking. Let us consider a pulse with arbitrary polarization, with complex amplitudes $\tilde{\mathcal{E}}_x(t)$ and $\tilde{\mathcal{E}}_y(t)$ along the principal axis characterized by the unit vectors $\hat{x}$ and $\hat{y}$:

$$\mathbf{E} = \frac{1}{2}\left(\hat{x}\tilde{\mathcal{E}}_{0x}(t) + \hat{y}\tilde{\mathcal{E}}_{0y}(t)\right)e^{i(\omega_\ell t - k_\ell z)} + c.c.. \tag{7}$$

The propagation of such a field through the nonlinear material leads to a coupling of the two polarization components. One can calculate, see [1], the nonlinear index change probed by polarizations along $\hat{x}$ and $\hat{y}$:

$$\begin{aligned}\Delta n_{\mathrm{nl},x} &= n_2\left[|\tilde{\mathcal{E}}_{0x}|^2 + \frac{2}{3}|\tilde{\mathcal{E}}_{0y}|^2\right]\\ \Delta n_{\mathrm{nl},y} &= n_2\left[|\tilde{\mathcal{E}}_{0y}|^2 + \frac{2}{3}|\tilde{\mathcal{E}}_{0x}|^2\right].\end{aligned} \tag{8}$$

In an element of thickness d_m, this induced birefringence leads to a phase change between the x and y components of the field vector

$$\Delta\Phi(t) = \frac{2\pi}{\lambda_\ell}\left(\Delta n_{\mathrm{nl},x} - \Delta n_{\mathrm{nl},y}\right) = \frac{2\pi n_2 d_m}{3\lambda_\ell}\left[|\tilde{\mathcal{E}}_{0x}(t)|^2 - |\tilde{\mathcal{E}}_{0y}(t)|^2.\right] \tag{9}$$

The phase shift is time dependent, and, in combination with another element, can represent an intensity-dependent loss element.

To illustrate this further let us consider a sequence of such a birefringent element and a linear polarizer. We assume that the incident pulse, $\mathcal{E}_0\cos(\omega t)$, is linearly polarized with components

$$\begin{aligned}\mathcal{E}_{0x}(t) &= \mathcal{E}_0(t)\cos\alpha\\ \mathcal{E}_{0y}(t) &= \mathcal{E}_0(t)\sin\alpha.\end{aligned} \tag{10}$$

The pass direction of the polarizer is at $\alpha + 90^o$ resulting in zero transmission through the sequence for low-intensity light ($\Delta\Phi \approx 0$). Neglecting a common phase the field components at the output of the nonlinear element are

$$\begin{aligned}\mathcal{E}'_x(t) &= \left[\mathcal{E}_0(t)\cos\alpha\right]\cos(\omega_\ell t)\\ \mathcal{E}'_y(t) &= \left[\mathcal{E}_0(t)\sin\alpha\right]\cos\left[\omega_\ell t + \Delta\Phi(t)\right].\end{aligned} \tag{11}$$

Next the pulse passes through the linear polarizer. The total transmitted field is the sum of the components from $\mathcal{E}'_x(t)$ and $\mathcal{E}'_y(t)$ along the polarizer's path direction

$$\mathcal{E}_{out}(t) = \mathcal{E}_0(t)\cos\alpha\sin\alpha\left\{\cos(\omega_\ell t) + \cos\left[\omega_\ell t + \Delta\Phi(t)\right]\right\} \tag{12}$$

The total output intensity $I_{out}(t) = \langle\mathcal{E}^2(t)\rangle$ is

$$I_{out}(t) = I_{in}(t)\frac{1}{2}\left[1 - \cos\Delta\Phi(t)\right]\sin^2(2\alpha). \tag{13}$$

Let us now assume a Gaussian input pulse $I_{in} = I_0\exp\left[-2(t/\tau_G)^2\right]$ and parameters of the nonlinear element so that for the pulse center the phase difference

$$\Delta\Phi(t=0) = \frac{2\pi n_2 d_m}{3\lambda_\ell}\mathcal{E}_0^2(t=0)\left(\sin^2\alpha - \cos^2\alpha\right) = \pi. \tag{14}$$

For this situation we obtain a transmitted pulse

$$I_{out}(t) = \frac{1}{2}I_{in}(t)\left\{1 - \cos\left[\pi e^{-2(t/\tau_G)^2}\right]\right\}. \tag{15}$$

The transmission is maximum (= 1) where the nonlinear element acts like a half-wave plate that rotates the polarization by 90^o, lining it up with the pass direction of the polarizer. For the parameters chosen here this happens at the pulse center (t = 0). The phase shift $\Delta\Phi$ is smaller away from the pulse center producing elliptically polarized output and an overall transmission that approaches zero in the pulse wings. Thus this sequence of elements can give rise to an intensity dependent transmission similar to a fast absorber.

References

[1] G. P. Agrawal. *Nonlinear Fiber Optics*. Academic Press, ISBN 0-12-045142-5, Boston, 1995.