1D Dielectric waveguide: the one D approach to fiber

We consider here the simplest form of a dielectric waveguide: a dielectric slab of infinite transverse dimension immersed in an infinite medium of slightly lower index.

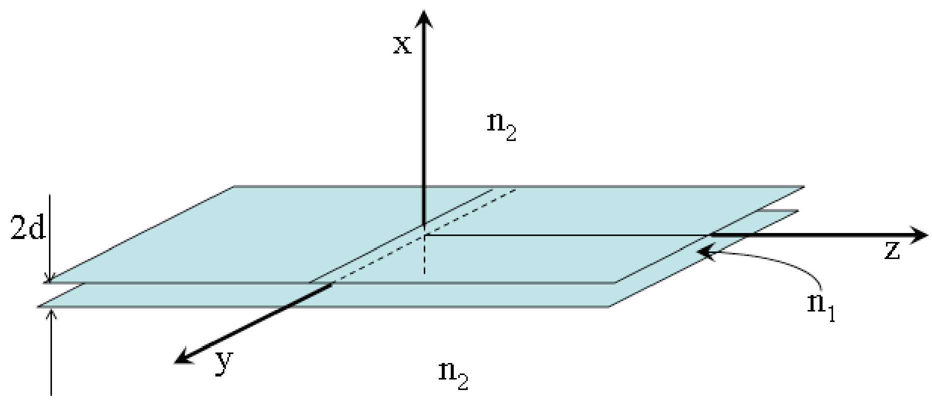

Figure 1: The boundaries are assumed to be infinite in the y and z directions.

Most waveguide study use Helmholz equation. From Maxwell's equation:

$$\Delta E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

with

$$E = Ee^{i\omega t}$$

$$\Delta E + \frac{n^2\omega^2}{c^2}E = 0$$

or

$$\Delta E + k^2 E = 0$$

or

$$\Delta E + n^2 k_0^2 E = 0$$

The plane wave (no dependence in x or y) solution is:

$$E = \mathcal{E}_0 e^{-ikz}$$

For a guided wave, we can insert in Helmholtz equation $E = \mathcal{E}(x, y)\exp(-i\beta z)$, to get

$$\nabla_t^2 \mathcal{E} + (k^2 - \beta^2)\mathcal{E} = 0$$

A "mode" will have a phase velocity ω/β, and a group velocity $d\omega/d\beta$.

Ray approach

Notations: θ_1 internal incidence angle, θ_2 external incidence angle.

Upward ray:

$$E_u = \mathcal{E}_0 e^{-ik_{x,1}x} e^{-i\beta z}$$

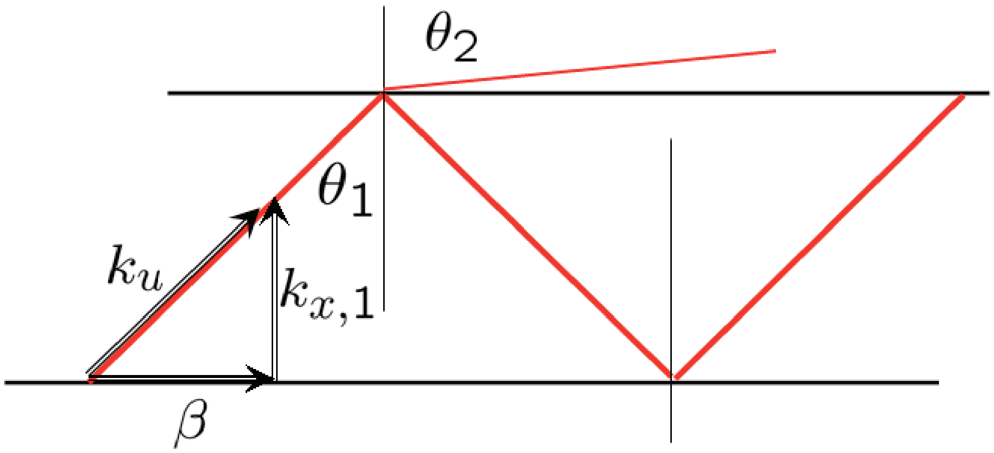

Figure 2: .

"Leaky modes": there is some light exiting. Fresnel equations give the amplitudes of the reflected and transmitted waves, different for TE and TM modes (s or p polarization).
Proper guiding: TIR at the interface, $\theta_1 > \theta_c$. Fresnel equations give the phases of the reflected wave, different for TE and TM modes (s or p polarization), the reflected amplitude being 1.

In medium 2:

$$E_2 = \mathcal{E}_{20} e^{-ik_{x,2}x} e^{-i\beta z} \to \mathcal{E}_{20} e^{-\gamma_2 x} e^{-i\beta z}$$

γ_2 comes from Snell's law:

$$\gamma_2 = ik_{x,2} = in_2 k_0 (-i) \sqrt{(\frac{n_1}{n_2})^2 \sin^2\theta_1 - 1}$$

Reflection coefficients TE

$$\Gamma_{TE} = \frac{k_{x,1} - k_{x,2}}{k_{x,1} + k_{x,2}} = \frac{k_{x,1} + i\gamma_2}{k_{x,1} - i\gamma_2} = \frac{Z}{Z^*} = e^{2i\phi_{TE}}$$

After two reflection and an up and down path, the total phase shift should be $2m\pi$:

$$4\phi - 2k_{x,1}d = 2m\pi$$

or

$$\phi = \arctan \gamma_2/k_{1,x} = k_{x,1}d/2 + m\pi/2$$

which leads to the condition for m = 0, 2, 4:

$$\boxed{\gamma_2 d = k_{x,1}d \tan k_{x,1}d/2} \tag{1}$$

and, for $m = 1, 3, 5...$

$$\boxed{\gamma_2 d = k_{x,1}d \frac{1}{\tan k_{x,1}d/2}} \tag{2}$$

To find the modes: combine with

$$\gamma_2^2 = \beta^2 - n_2^2 k_0^2 \tag{3}$$

$$k_{x,1}^2 = n_1^2 k_0^2 - \beta^2 \tag{4}$$

to get the equation of a circle:

$$(\gamma_2 d)^2 + (k_{x,1}d)^2 = k_0^2 d^2 (n_1^2 - n_2^2)$$

The right hand side of this expression is the radius R of a circle. This dimensionless radius:

$$\boxed{R = k_0 d\sqrt{n_1^2 - n_2^2}} \tag{5}$$

is the most important parameter in a waveguide. For the modes $m = 1, 3, 5...$, the possible solutions for $k_{x,1}$, γ_2, are at the intersection of the circle of radius R, with the curves given by Eq. (1). There is only one intersection if $R \leq \pi$, which correspond to a symmetric (cos) mode. At $R \geq \pi$, an intersection with the cotan is possible, corresponding to $m = 1$. This corresponds to the antisymmetric mode (sin).

Reflection coefficients TM

$$\Gamma_{TE} = \frac{n_2^2 k_{x,1} - n_1^2 k_{x,2}}{n_2^2 k_{x,1} + n_1^2 k_{x,2}} = \frac{n_2^2 k_{x,1} + i n_1^2 \gamma_2}{n_2^2 k_{x,1} - i n_1^2 \gamma_2} = \frac{Z}{Z^*} = e^{2i\phi_{TM}}$$

Same derivation as in TE to find the modes.

Graphic solution

The graphical representation of the equations for the modes of the plane dielectric wavequide is given in Fig. 3. The values of γ_2 and k_{x1} are inserted

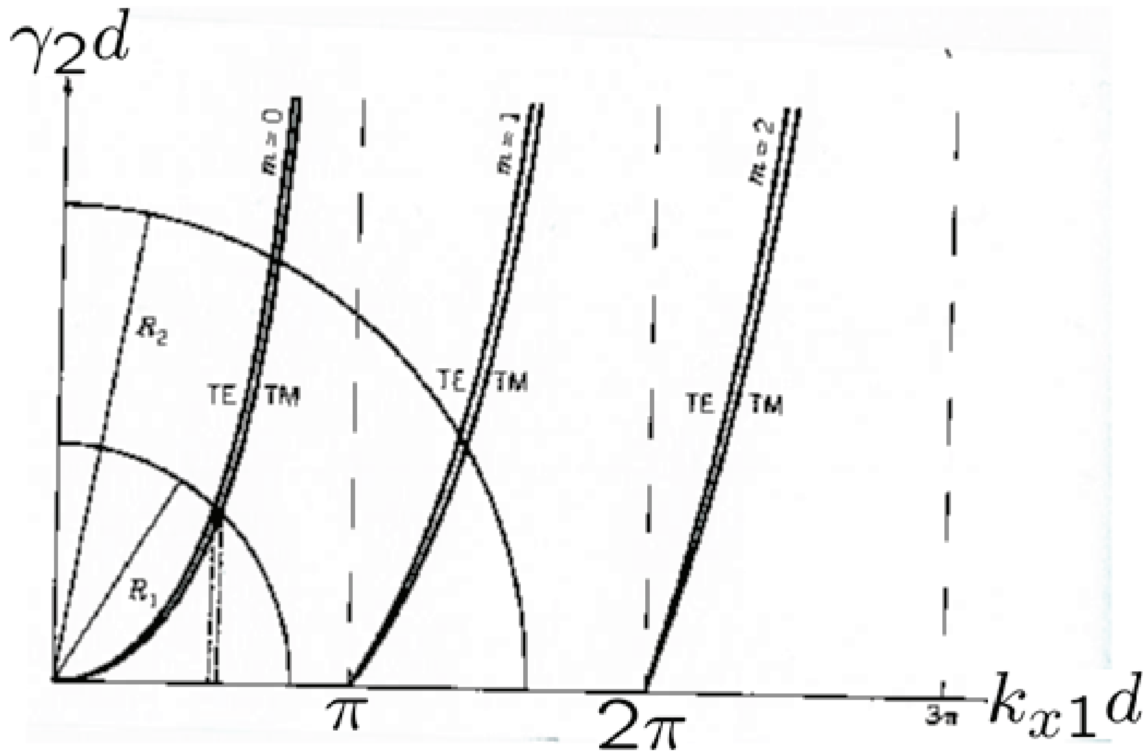

Figure 3: Solutions for the mode parameters are at the intersection of the circle of radius R and the $x \tan x$ alternating with the x cotan x curves.

in Eq. (3) or (4) to find the waveguide propagation vector β.

Fiber

The derivation of the modes for a step index fiber is much more complex than for the slab waveguide. Details can be found in "Fundamentals of Optical Fibers" by John Buck (Wiley). For a step index fiber, the important parameter is the "V" parameter:

$$\boxed{V = k_0 a \sqrt{n_1^2 - n_2^2}} \tag{6}$$

where a vis the radius of the sphere. In one dimension, fields confined by a boundary condition are generally associated with sin and cos functions. In cylindrical symmetry, one deals generally with Bessel functions. So it should not be surprising that the condition $R \leq \pi$ (the first zero of the sine function) is replaced by $V \leq 2.405$, the first zero of the Bessel function $J_0(V)$. In the slab waveguide as in the fiber, single mode implies

- small index difference
- long wavelength
- small size a

Definitions for step index fibers

Core radius a
Normalized index difference (expressed in percent) $\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1} = \frac{\delta n}{n_1}$.
Numerical aperture $NA = \sqrt{n_1^2 - n_2^2} = n_1\sqrt{2\Delta}$
The numerical aperture is the external half angle corresponding to the critical angle in the fiber.
Typical single mode: $\Delta = 0.2\%$.
Typical multimode: $\Delta = 1\%$.
Cladding 125 μm.