**Fresnel Rhomb calculations.**

The angle of incidence (in the glass of index $n$) is $\theta$. On the other side of the interface, the index is 1, and the angle is $\theta_2$ which is given by Snell's law:

$$n_2 \sin\theta_2 = \sin\theta_2 = n \sin\theta \tag{1}$$

and

$$\cos\theta_2 = \sqrt{1 - n^2 \sin^2\theta} = i\sqrt{n^2 \sin^2\theta - 1} \tag{2}$$

This implies that you have to replace $\sin\theta_2$ by $n\sin\theta$ and $\cos\theta_2$ by $i\sqrt{n^2\sin^2\theta - 1}$ in Fresnel equations, which become then complex numbers. The two Fresnel equations have then the form:

$$r_\parallel = \frac{\cos(\theta) - n\cos\theta_2}{\cos(\theta) + n\cos\theta_2} = \frac{a - ib}{a + ib} = \frac{(a^2 - b^2) - 2iab}{a^2 + b^2}, \tag{3}$$

$$r_\perp = \frac{n\cos(\theta) - \cos\theta_2}{n\cos(\theta) + \cos\theta_2} = \frac{c - id}{c + id} = \frac{(c^2 - d^2) - 2icd}{c^2 + d^2} \tag{4}$$

It is easy to verify that $|r|^2 = 1$ for both parallel and perpendicular condition, independently of the angle provided $\theta \geq \theta_{\text{critical}}$

The important quantities for our problem are

$$\tan\phi_\parallel = \frac{-2ab}{a^2 - b^2}$$

$$\tan\phi_\perp = \frac{-2cd}{c^2 - d^2}$$

If after two reflections, a linear polarization should become circular, that means a difference of phase of 90 degrees ($\lambda/4 \equiv 2\pi/4 = \pi/2$). There are two obvious ways to proceed:

1. compute $\tan(\phi_\parallel - \phi_\perp) = 1$
2. compute $\tan(2\phi_\parallel - 2\phi_\perp) = \infty$

These approaches are "straightforward but tedious. There is a simpler approach if one notes that Eqs. (3) and (4) is the ratio of a complex number and its complex conjugate $z/z*$, and thus the phase $\phi = 2\delta$ where $\delta$ is the phase shift of $z$. Therefore a third simple approach is to compute $\tan(\delta_\parallel - \delta_\perp) = \tan \pm\pi/8 = \pm 0.41421$ We have:

$$\tan\delta_\parallel = \frac{-n\sqrt{n^2\sin^2\theta - 1}}{\cos\theta}$$

$$\tan\delta_\perp = \frac{-\sqrt{n^2\sin^2\theta - 1}}{n\cos\theta}$$

The tangent of the difference angle is:

$$
\begin{aligned}
\tan(\delta_\parallel - \delta_\perp) &= \frac{\tan\delta_\parallel - \tan\delta_\perp}{1 + \tan\delta_\perp \tan\delta_\parallel} \\
&= \frac{-\cos\theta\sqrt{n^2\sin^2\theta - 1}}{n\sin^2\theta} \\
&= \frac{-\sqrt{n^2 - 2}}{n} \qquad (5)
\end{aligned}
$$

where the last equation (5) is for the value $\theta = \pi/4$. Taking the square of Eq. (5) yields a simple second order equation in $n$, which has as solution

$$
n = \sqrt{\frac{-2}{t^2 - 1}} = 1.554, \qquad (6)
$$

where $t = \tan\pi/8 = 0.4142$.