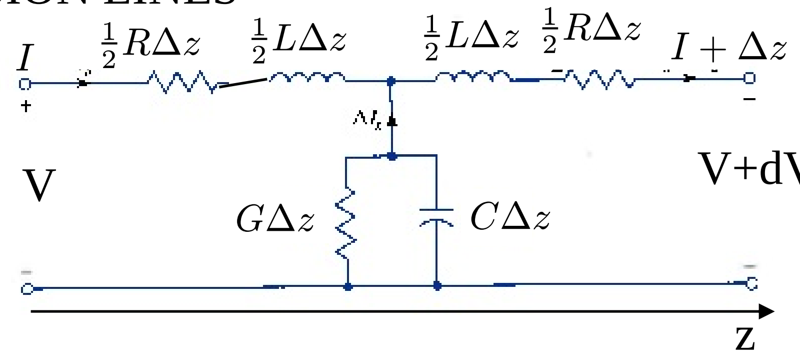
From transmission line to waveguide to fibers

TRANSMISSION LINES

Free space

$$\left(\Delta_{tr} - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathcal{E}e^{i(\omega t)} = 0$$

$$\left(\Delta_{tr} + k^2\right)\mathcal{E} = 0$$



$$V = (\frac{1}{2}R\Delta z + \frac{i}{2}\omega L)I + (\frac{1}{2}R\Delta z + \frac{i}{2}\omega L)(I+\Delta I) + (V + \Delta V)$$

$$\Delta I = -(G + i\omega C)V\Delta z$$

$$\nabla \times H \rightarrow \frac{dH_y}{dz} = -(\sigma + i\omega\epsilon')E_x \qquad \frac{dI}{dz} = -(G + i\omega C)V$$

$$\nabla \times E \rightarrow \frac{dE_x}{dz} = -i\omega\mu H_y \qquad \frac{dV}{dz} = -(R + i\omega L)I$$

$$\eta = \frac{E_x}{H_y} = \sqrt{\frac{\mu}{\epsilon'}} \qquad \eta = \frac{V}{I} = \sqrt{\frac{R + i\omega L}{G + i\omega C}} \rightarrow \sqrt{\frac{L}{C}}$$

$$k = \omega\sqrt{\epsilon'\mu} \qquad \beta = \omega\sqrt{LC}$$

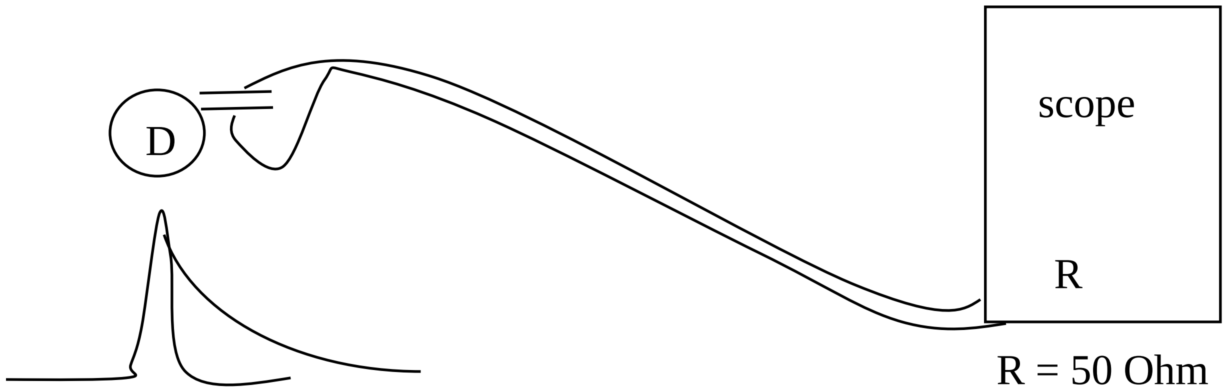
$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon'\mu}} \qquad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Coaxial line $$C = \frac{2\pi\epsilon'}{\ln\frac{b}{a}} \qquad L = \frac{\mu}{2\pi}\ln\frac{b}{a}$$

Two wires diameter *a* spaced by *d* $$C = \frac{\pi\epsilon'}{\ln\frac{d}{a}} \qquad L = \frac{\mu}{\pi}\ln\frac{d}{a}$$

Planar: width *b* spaced by *d* $$C = \frac{\epsilon' b}{d} \qquad L = \frac{\mu d}{b}$$

Impedance $$\sqrt{\frac{L}{C}} \qquad \sqrt{\frac{\mu}{\epsilon}} \times \ \ldots$$



R = 50 Ohm

R = 1MOhm

RC

V = IR

WAVEGUIDES – Eigenvalue – eigenfunction approach.

Most waveguide study use Helmholz equation. From Maxwell's equation:

$$\Delta E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

with

$$E = Ee^{i\omega t}$$

$$\Delta E + \frac{n^2\omega^2}{c^2}E = 0$$

or

$$\Delta E + k^2 E = 0$$

or

$$\Delta E + n^2 k_0^2 E = 0$$

The plane wave (no dependence in x or y) solution is:

$$E = \mathcal{E}_0 e^{-ikz}$$

For a guided wave, we can insert in Helmholtz equation $E = \mathcal{E}(x,y)\exp(-i\beta z)$, to get

$$\nabla_t^2\mathcal{E} + (k^2 - \beta^2)\mathcal{E} = 0$$

$$-\beta^2\mathcal{E} + \nabla_t^2\mathcal{E} + k^2\mathcal{E} = 0$$

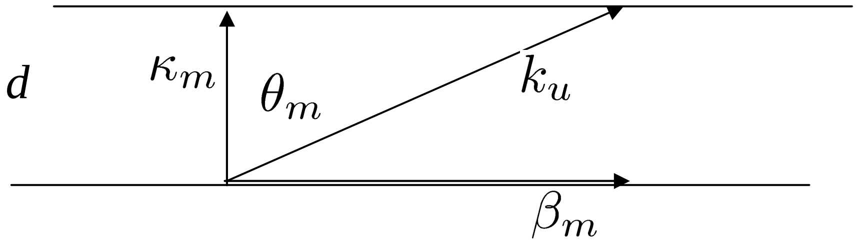
A "mode" will have a phase velocity ω/β, and a group velocity $d\omega/d\beta$.

$$E = \mathcal{E}(x,y)e^{i(\omega t - \beta z)}$$

Remember something like **Hψ = Eψ** ?

METALLIC WAVEGUIDE

$$\kappa_m d + \phi + \kappa_m d + \phi = 2m\pi$$



E ⊙ TE: ϕ=0

H ⊙ TM: ϕ=π

$$\theta_m = \arccos(\frac{m\pi}{d}\frac{\lambda}{2\pi n}) = \arccos(\frac{m\lambda}{2nd})$$

$$\kappa_m d = 2m\pi/2d$$

$$k = \Omega n/c$$

$$\beta_m = \sqrt{k^2 - \kappa_m^2} = k\sqrt{1 - (\frac{m\pi c}{\Omega n d})^2} = \frac{n\Omega}{c}\sqrt{1 - (\frac{\omega_{cutoff}}{\Omega})^2}$$

$$\omega_{cutoff} = \frac{m\pi c}{nd}$$

Phase velocity: $v_p = \frac{\Omega}{\beta} = \frac{\Omega}{k\sin\theta_m}$

Group velocity: $\frac{1}{v_g} = \frac{d\beta}{d\Omega}$ $\qquad v_g = \frac{c}{n}\sin\theta_m$

CO_2 laser: 10.6 μm

$c/\lambda = 3\ 10^{13}$ Hz

$$\omega_{cutoff} = \frac{m\pi c}{nd}$$

Frequency below cutoff: β imaginary → attenuation

$$\nu_{cutoff} = \frac{mc}{2nd}$$

$$\nu > \nu_{cutoff}$$

$$\lambda < \lambda_{cutoff} = \frac{2nd}{m}$$

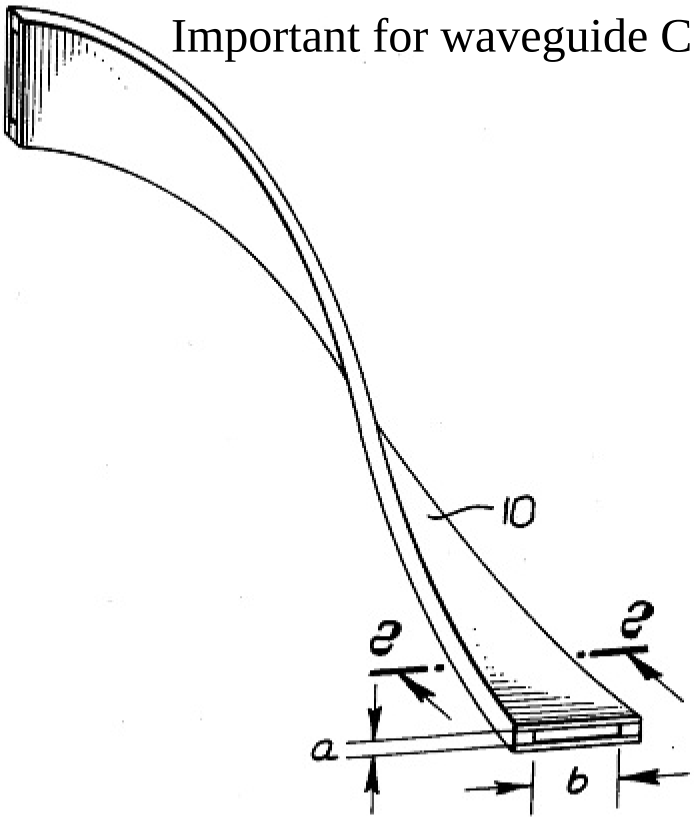
d = 5 μm

Order m = 1, cutoff 10 micron

Order m = 2, cutoff 5 micron

So 10 micron light will not pass in second order.

Important for waveguide CO_2 lasers



Propagation of infrared light in flexible hollow waveguides

E. Garmire, T. McMahon, and M. Bass

January 1976 / Vol. 15, No. 1 / APPLIED OPTICS 145

350-96.12 $$$ SR

E197Y OR 4,068,920

X350-96.30

United State[s Patent] [1]] **4,068,920**

Bass et al. [4]5] **Jan. 17, 1978**

[54] **FLEXIBLE WAVE GUIDE FOR LASER LIGHT TRANSMISSION**

[75] Inventors: **Michael Bass,** Pacific Palisades; **Elsa Garmire,** Pasadena; **Thomas R. McMahon,** Los Angeles, all of Calif.

[73] Assignee: **University of Southern California,** Los Angeles, Calif.

[21] Appl. No.: **716,296**

[22] Filed: **Aug. 20, 1976**

[51] **Int. Cl.**[2] **G02B 5/14**

[52] **U.S. Cl.** .... **350/96 WG**

[58] **Field of Search** .... 350/96 WG; 333/95 R, 333/95 A; 331/94.5 C

[56] **References Cited**

U.S. PATENT DOCUMENTS

3,583,786	6/1971	Marcatili	350/96 WG
3,939,439	2/1976	Fletcher et al.	350/96 WG X

OTHER PUBLICATIONS

E. A. J. Marcatili, R. A. Schmeltzer, Bell Syst. Tech. Journ. No. 43, p. 1783, 1964.

H. Nishihara, T. Inoue, J. Koyama, "Low-Loss Parallel-Plate Waveguide at 10.6 μm", Appl. Phys. Lett vol. 25, No. 7, Oct. 1974, pp. 391-393.

K. D. Lackman, W. H. Steier, "Waveguides: Characteristic Modes of Hollow Rectangular Dielectric Waveguides", Appl. Opt. vol. 15, No. 5, May 1976, pp. 1334-1340.

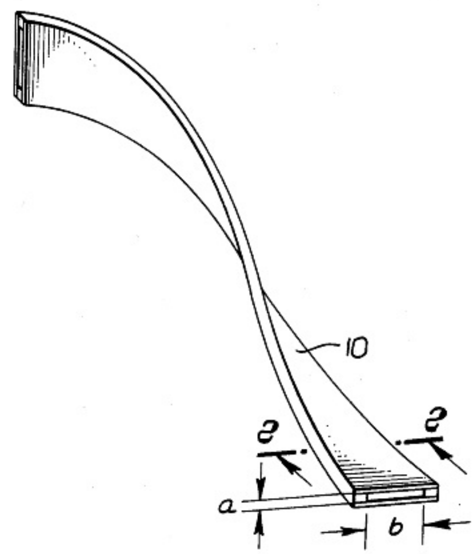
R. L. Abrahms, W. B. Bridges, "Characteristic of Sealed-Off Waveguide CO_2 Laser", IEEE J of Quantum Elec. vol. QE-9, No. 9, Sept. 1973, pp. 940-946.

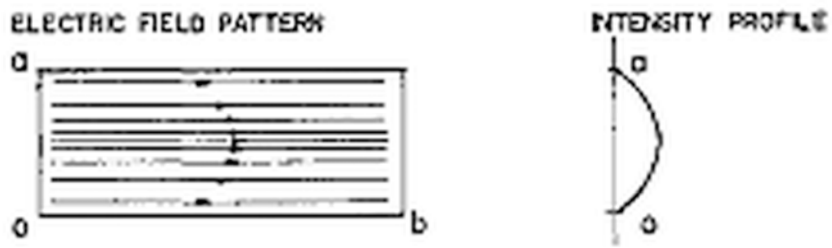
Primary Examiner—Paul A. Sacher
Assistant Examiner—Rolf Hille
Attorney, Agent, or Firm—Harris, Kern, Wallen & Tinsley

[57] **ABSTRACT**

A flexible hollow rectangular wave guide for transmission of radiation in the infrared portion of the spectrum, including infrared laser radiation. A wave guide which may be bent and twisted while providing low loss transmission of infrared radiation. An all metal wave guide with the width to heighth ratio at least 4 to 1. A wave guide with metal surfaces on the long dimension and dielectric surfaces on the short dimension and having a width to height ratio at least 2 to 1.

17 Claims, 12 Drawing Figures



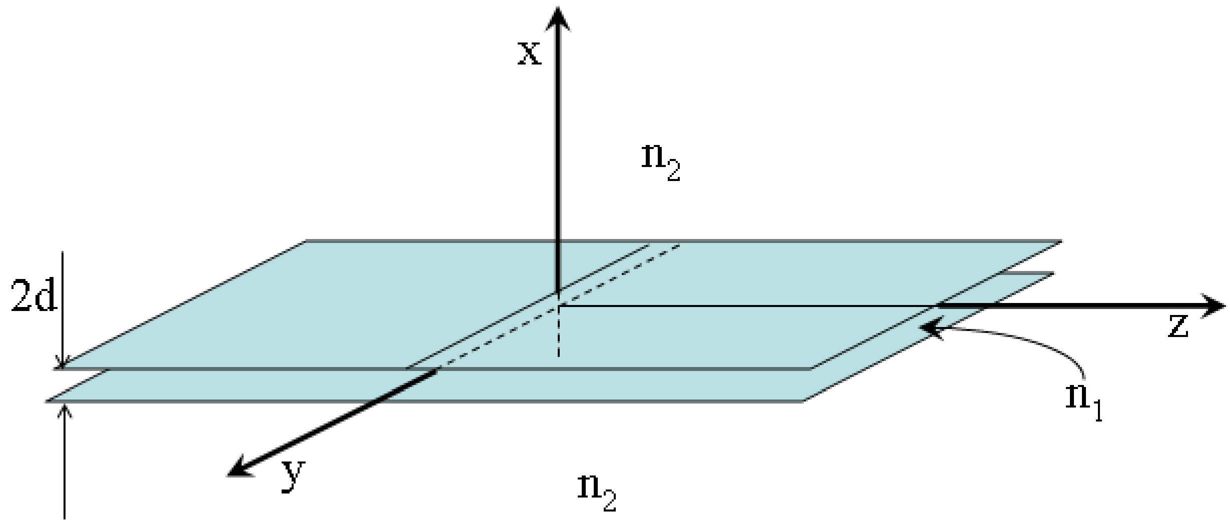


The polarization of the lowest loss mode in these long, thin multimode waveguides has an electric field parallel to the long dimension of the guide cross section. This is the opposite polarization from single-mode rectangular microwave guides. This difference is attributable to the complex dielectric constant in the ir case.

The microwave analysis with a real conductivity is not valid at 10.6,4m.

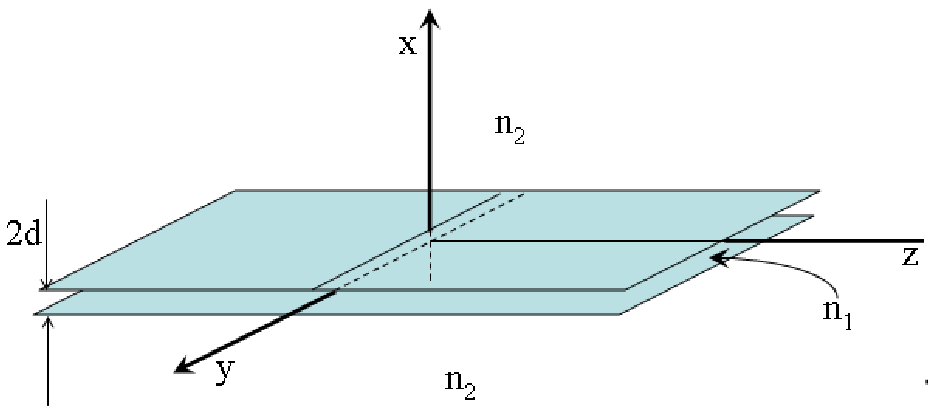
n, $\boldsymbol{K} >> 1$ a copper waveguide 100 ,m thick and 6 mm wide will transmit 95% of the incident light through a meter.

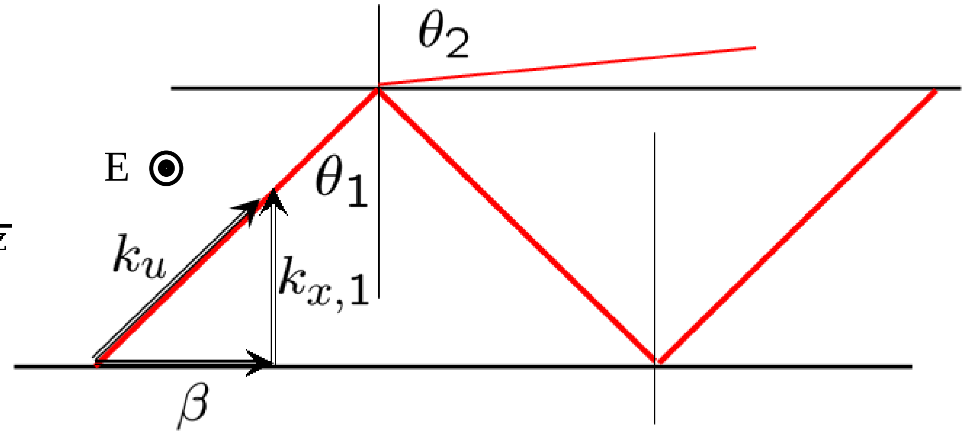
DIELECTRIC WAVEGUIDE



DIELECTRIC WAVEGUIDE

$$\kappa_m d + \phi + \kappa_m d + \phi = 2m\pi$$





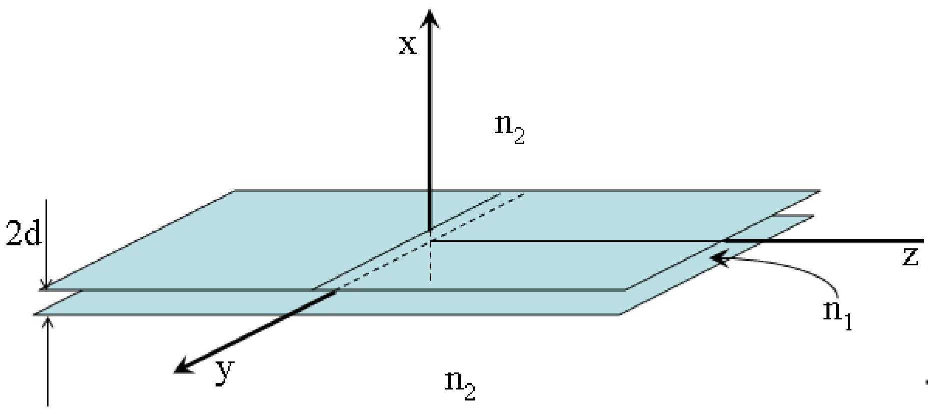
In medium 2: $E_2 = \mathcal{E}_{20} e^{-ik_{x,2}x} e^{-i\beta z} \rightarrow \mathcal{E}_{20} e^{-\gamma_2 x} e^{-i\beta z}$

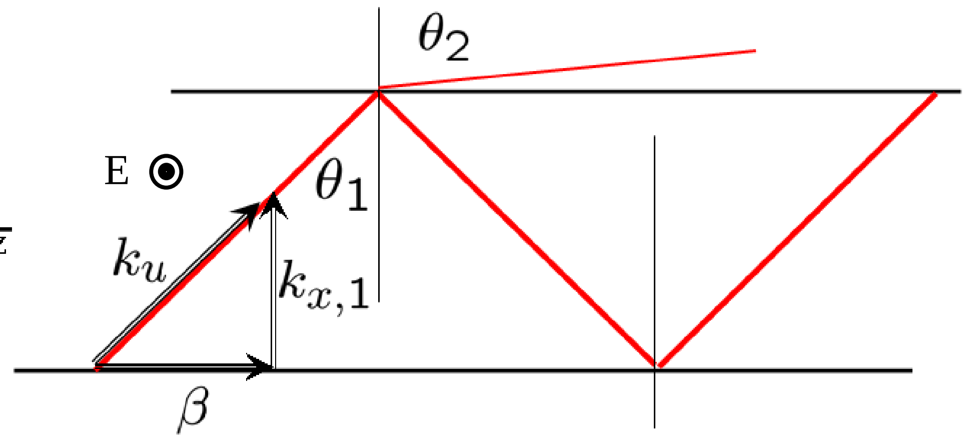
From Snell's law: $\gamma_2 = ik_{x,2} = in_2 k_0 (-i) \sqrt{(\frac{n_1}{n_2})^2 \sin^2\theta_1 - 1}$

Reflection coefficient TE: $\Gamma_{TE} = \frac{k_{x,1} - k_{x,2}}{k_{x,1} + k_{x,2}} = \frac{k_{x,1} + i\gamma_2}{k_{x,1} - i\gamma_2} = \frac{Z}{Z^*} = e^{2i\phi_{TE}}$

DIELECTRIC WAVEGUIDE

$$\kappa_m d + \phi + \kappa_m d + \phi = 2m\pi$$





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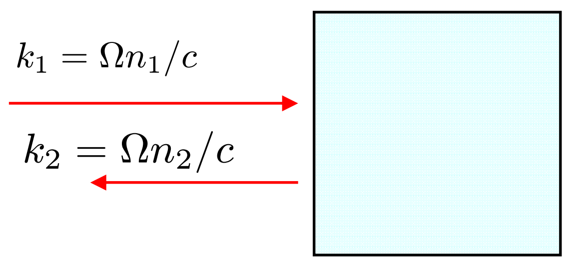


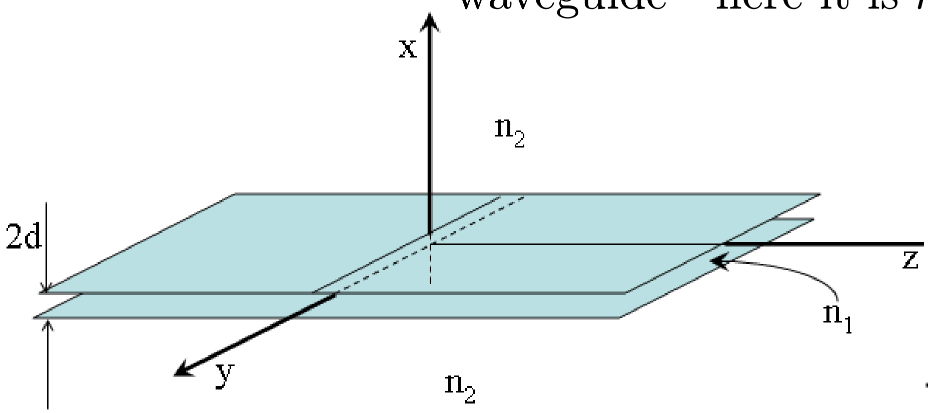
κ_m = notation for metallic waveguide - here it is $k_{x,1}$

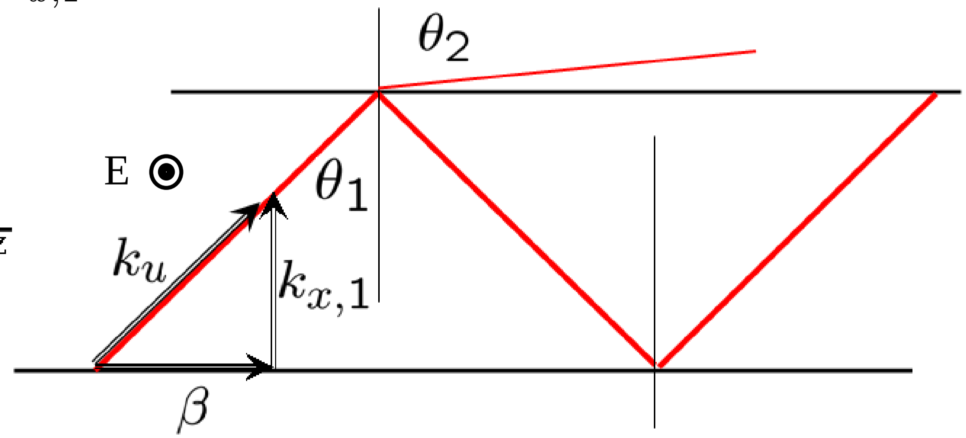
Field reflection: $\frac{n_1 - n_2}{n_1 + n_2} = \frac{k_1 - k_2}{k_1 + k_2}$

DIELECTRIC WAVEGUIDE

κ_m = notation for metallic waveguide - here it is $k_{x,1}$

$$\kappa_m d + \phi + \kappa_m d + \phi = 2m\pi$$





In medium 2: $$E_2 = \mathcal{E}_{20} e^{-ik_{x,2}x} e^{-i\beta z} \rightarrow \mathcal{E}_{20} e^{-\gamma_2 x} e^{-i\beta z}$$

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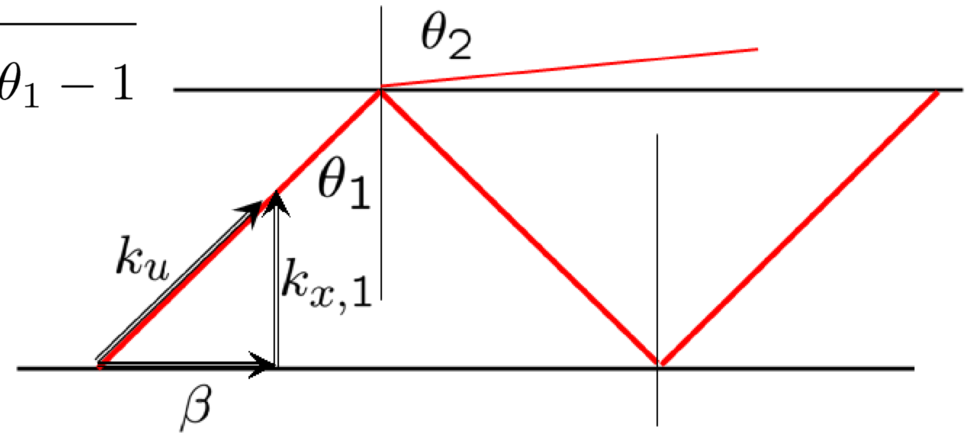
$$\kappa_m d + \phi + \kappa_m d + \phi = 2m\pi \rightarrow 4\phi - 2k_{x,1} d$$

$$\phi = \arctan \gamma_2 / k_{1,x} = k_{x,1} d/2 + m\pi/2$$

$$\gamma_2 d = k_{x,1} d \tan k_{x,1} d/2$$ For m= 0, 2, 4…

$$\gamma_2 = ik_{x,2} = in_2k_0(-i)\sqrt{(\frac{n_1}{n_2})^2 \sin^2\theta_1 - 1}$$

$$\gamma_2^2 = k_0^2 n_1^2 \sin^2\theta_1 - n_2^2 k_0^2$$



$$\begin{aligned} \gamma_2^2 &= \beta^2 - n_2^2 k_0^2 \\ k_{x,1}^2 &= n_1^2 k_0^2 - \beta^2 \end{aligned}$$

to get the equation of a circle:

$$(\gamma_2 d)^2 + (k_{x,1} d)^2 = k_0^2 d^2 (n_1^2 - n_2^2)$$

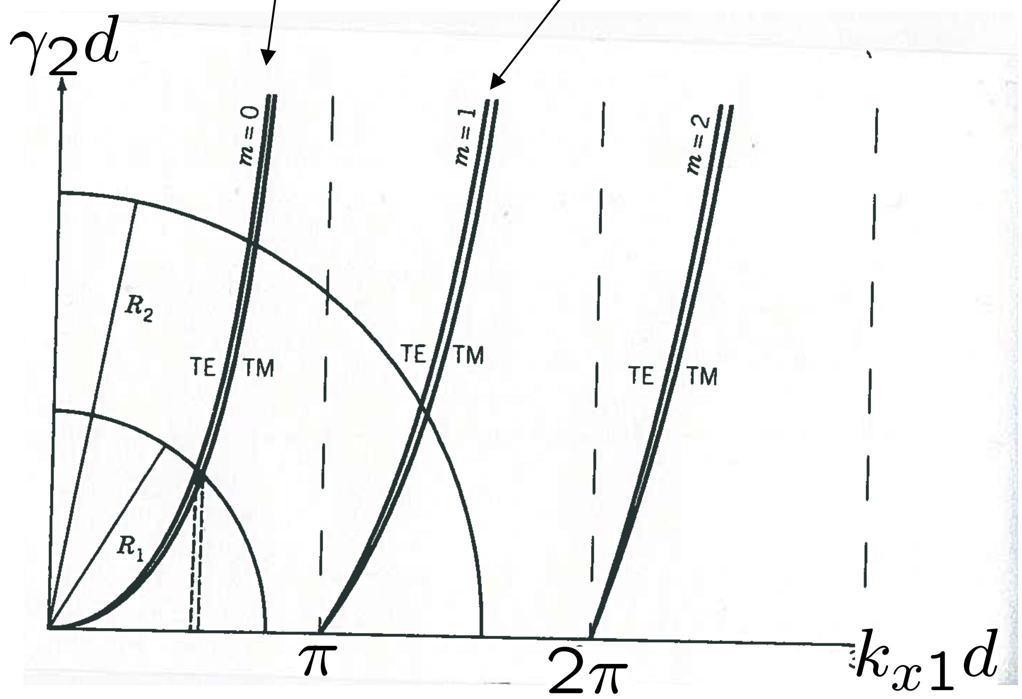
The right hand side of this expression is the radius R of a circle. Dimensionless radius:

$$\boxed{R = k_0 d\sqrt{n_1^2 - n_2^2}}$$

Slab waveguide solution

$$\gamma_2 d = k_{x,1} d \tan k_{x,1} d/2$$

$$\gamma_2 d = k_{x,1} d \frac{1}{\tan k_{x,1} d/2}$$



$$R = k_0 d \sqrt{n_1^2 - n_2^2}$$