Laser Physics (Physics 464)
Homework I, due Wednesday, September 4, 2024

Transverse Doppler shift

Year 202?, the Artemis NASA mission is on the moon with a high resolution wavemeter. Jan Hall's one micron wavelength laser with one Herz bandwidth is beamed at that detector. Is that sufficiently narrow bandwidth? If not, what is the minimum bandwidth required? How long do you have to track the detector on the moon with your laser beam in order to do this measurement? Assume a circular orbit of radius L_0 for the moon around the earth.

Solution

Answer
Radius of the moon orbit: about 1 s at the speed of light $\approx$ 300,000 km or $3 \cdot 10^8$ m (a good approximation of $3.48 \cdot 10^8$ m).
The angular velocity of the moon is $\Omega_R = 2\pi/(27.3 \text{ day})$, or in s^{-1}

$$\frac{2\pi}{27.3 \times 24 \times 60 \times 60} = 2.66 \cdot 10^{-6} \text{s}^{-1}$$

. The transverse velocity (velocity of the moon) is:

$$v = R\Omega = 3.48 \cdot 10^8 \times 2.66 \cdot 10^{-6} \approx 900 \text{m/s}.$$

The transverse Doppler shifted frequency is thus:

$$\nu_d = \nu_0 \sqrt{1 - \frac{v^2}{c^2}} = \nu_0 \left[\sqrt{1 - \frac{8 \cdot 10^5}{9 \cdot 10^{16}}} \right]$$

Try punching that on your calculator! It is simpler on the back of an envelope. The Doppler shift is:

$$\Delta\nu = \nu_d - \nu_0 = \nu_0 \times \left\{ 1 - \sqrt{1 - 9 \cdot 10^{-12}} \right\} \approx \nu_0 \approx \nu_0 \left[1 - (1 - 4.5 \cdot 10^{-12}) \right]$$

At the wavelength of 1 μm, $\nu_0 = 3 \cdot 10^{14}$ s^{-1}, and $\Delta\nu_d = 3 \cdot 10^{14} \times 4.5 \cdot 10^{-12} =$ 1.35 kHz Conclusion: easily measurable. Measurement time > 1 ms.

Longitudinal Doppler shift

The orbit of the moon is not circular, but has a fairly large eccentricity of 13%. The moon-earth distance varies periodically. Take that variation to be sinusoidal. What is the Doppler shift at the point(s) where the longitudinal component of the velocity is maximum?
The earth is at a focus of the ellipse. If e is the eccentricity, s the shortest distance, the longest distance is $e+s$ The amplitude of the distance oscillation is $e = 0.13R$.

Solution

Answer - the earth is at the focus of the ellipse
If e is the eccentricity, s the shortest distance, the longest distance is $e + s$ The amplitude of the distance oscillation is $e = 0.13R$. The longitudinal velocity is $dd/dt = 0.13R\Omega \sin \Omega t$ with the maximum value of $0.13R\Omega$.
Interesting that the wrong approach gives the right Doppler shift. Note the frequency (of the periodicity) doubling with the source in the center rather than at the focus of the ellipse.

The duckling transverse Doppler shift

Find an expression for the transverse Doppler shift of the duckling (see class notes).

Solution

See ppt file Duck-Doppler