Nonlinear Optics 2026 — Homework 1

Due Wednesday, February 4, 2026

electron trajectories

An electron is placed at $z = 0$, $t = t_0$ in a laser field at 800nm, polarized along z, of intensity $5 \cdot 10^{14}\text{W/cm}^2$.

The laser field is $E = \mathcal{E} \cos \omega t$. Write the equation of motion for the electron. Integrate, to find its velocity $v(t)$ and position $z(t)$. Take t_0 to be such that $\omega t_0 = 0.1$. Find the time(s) t_f (if any) at which the electron position is again $z(t_f) = 0$. You may make a small angle approximation. What is the electron energy at that point (in eV)?

Equation of motion:

$$\frac{dv}{dt} = -\frac{eE}{m} \cos \omega t. \tag{1}$$

Integrating:

$$v(t) - 0 = -\frac{eE}{m\omega} \left[\sin(\omega t) - \sin(\omega t_0)\right] = \frac{dz}{dt}. \tag{2}$$

Integrating again to find the trajectory:

$$z(t) - 0 = \frac{eE}{m\omega^2} \left[\cos(\omega t) - \cos(\omega t_0) + \sin(\omega t_0)(\omega t - \omega t_0)\right]. \tag{3}$$

If we start at $\omega t_0 = 0$:

$$z(t) = \frac{eE}{m\omega^2} \left[\cos(\omega t) - 1\right].$$

The trajectory is in the negative z, the electron reaches the farthest distance at $\omega t = \pi$, to return to $z = 0$ at $\omega t_1 = 2\pi$.We can therefore make the approximation $\omega t_1 \approx 2\pi + \phi_1$. Substituting:

$$z(t_1) = 0 = \frac{eE}{m\omega^2} \left[-\frac{\phi_1^2}{2} + \frac{\phi_0^2}{2} - \phi_0)(2\pi + \phi_1 - \phi_0) \right]. \tag{4}$$

which leads to a simple quadratic equation:

$$\frac{\phi_1^2}{2} - \phi_0\phi_1 + \frac{\phi_0^2}{2} - 2\pi\phi_0) = 0 \tag{5}$$

Substitute $\phi_0 = 0.1$, and you find 2 solutions: $\phi_1 = 1.22$ and $\phi_1 = 1.02$.

MURPHY'LAW STRIKES AGAIN!!!
This is incompatible with the small ϕ_1 assumption!

We have to solve Eq. (3) with $\cos \phi_0 = 0.995$:

$$\cos \phi_1 + 0.1\Phi_1 = 0.995 - 0.01 = 0.985.$$

A little bit of trial and error and we find $\phi_1 = 5.2$.
Substitute in Eq. (2) to find the velocity:

$$v(t) - 0 = -\frac{e\mathcal{E}}{m\omega}[-0.88 - 0.1] = 0.98\frac{e\mathcal{E}}{m\omega}$$

Kinetic energy in Joules:

$$mv^2/2 = \frac{e^2\mathcal{E}^2}{2m\omega^2} = \frac{e^2(2 \times 377I)\lambda^2}{8m\pi^2c^2}$$

Kinetic energy in eV:

$$mv^2/2 = \frac{e(2 \times 377I)\lambda^2}{8.m\pi^2c^2}$$

Result: $59 \times .98 = 58$ eV.