Nonlinear Optics 2025 — Homework 3

Due Wednesday, March 11, 2026

Consider the situation relating to the interaction of two counter-propagating pulses in a nonlinear medium characterized by a third order nonlinear susceptibility $\chi^{(3)}$. The purpose of this problem is to compare the counter-propagating interaction in the case of the homogeneous medium with the case of a set of Multiple Quantum Wells (MQW's) separated by a wavelength, as sketched in Fig. 1.

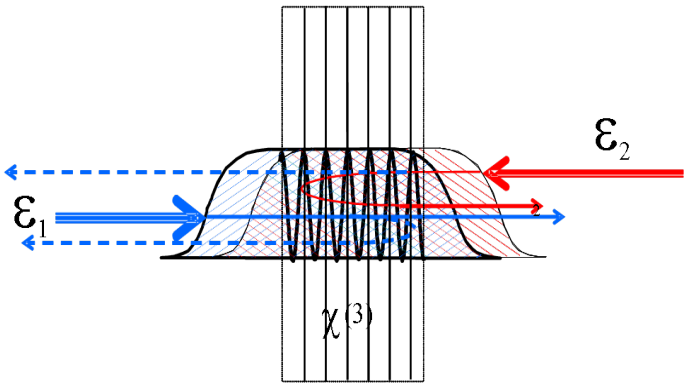

Figure 1: Geometry of counter-propagating wave interaction in a medium with a third order nonlinear susceptibility concentrated in quantum wells. We assume the spacing between quantum wells to have a negligible nonlinear susceptibility. The quantum wells are located at the antinodes of the standing wave field, resulting in a maximum interaction between the nonlinear medium and the light field.

For the homogeneous medium, the nonlinear susceptibility is uniform and equal to $\chi^{(3)}$. The stratified medium is assumed to be made of N (infinitely thin) quantum wells separated by half a wavelength, each quantum well having a susceptibility $\chi^{(3)}\delta(kz - j\pi)$. The medium susceptibility $\chi^{(NL)}$ can thus be represented by:

$$\chi^{(NL)}(z) = \chi^{(3)}\delta(kz - j\pi) \tag{1}$$

It should be remembered that the polarization represents en ensemble of dipoles induced by the radiation. Each of these dipole radiate, and it is the destructive interference of this radiation with the applied field that gives rise to absorption. This problem is already non-trivial in the case of linear polarization, hence you may appreciate that there is a notch of complexity added with nonlinear polarization. To appreciate the problem, you may recall the difference between a layered gain structure, and a homogeneous gain medium. In the case of the layered gain medium, the emission that is stimulated at each gain layer adds in phase in the forward *and backward* directions, if the layers are spaced by a half wavelength. For the homogeneous gain medium, the radiation emitted in the backward direction averages to zero.