Laser Physics I — Homework 4
Due Wednesday, October 18, 2023

Dye laser

The gain medium of a dye laser is a flowing dye jet. This is the gain medium with the largest damage threshold, since it is continuously replenished. It can be considered as a three level system, pumped from the ground state (which is also the lower lasing level) to a group of upper levels, from which there is a very fast (near instantaneous as compared to all other time constants) relaxation to the upper lasing level. The pump is provided by green laser (up to 20 W), focused onto a 200 micron thick jet. The following parameters are given:

- gain molecule concentration: $6 \cdot 10^{17}$ cm^{-3}
- relaxation time of upper lasing level: $T_1 = 2.5$ ns
- cross section of the focused beam: 30 μm^2 (which is also the cross section of the waist of the cavity located in the jet)
- Absorption cross section of the pump beam $\sigma_p = 1. \cdot 10^{-16}$ cm^2
- Gain cross section (for the lasing beam): $\sigma = 5. \cdot 10^{-16}$ cm^2
- Pump wavelength 532 nm
- Lasing wavelength 590 nm
- Reflectivity of the output mirror: 80%

Find the pump power required to have zero gain/zero absorption

Hint: use the two-level rate equations for the population difference modified to represent a three level system. The zero gain condition defines the initial population difference. The equilibrium condition leads to R'. Backtrack the changes in variable to arrive at the pumping rate R and the pump intensity I_p.

Find the pump power required for threshold

Find the gain required to compensate the loss (due only to transmission through the output coupler). Given the gain and cross-section leads to the inversion ΔN_{eq}. From there the procedure for finding the pump power is the same as above.

What is the output power at a pump power of 6 W?

Hint: Find the pump intensity, then the pump rate R. Follow the changes in variable to find successively T_p, I'_s and R', which leads to the equilibrium inversion ΔN_e. The condition that this inversion saturated to the threshold value (previous question) leads to the intensity.

$$\begin{aligned}
\Delta N_{eq} &= \Delta N_0 + R' T_p = 0 \\
R' &= -R\Delta N_0 \\
RT_p &= 1 \\
R &= \frac{1}{T_p} = \frac{1}{T_1} + \frac{R}{2} \\
R &= \frac{2}{T_1} = 0.8 \cdot 10^9 \mathrm{s}^-1 = \sigma_p I_p/(h\nu_p) \\
I_p &= \frac{Rh\nu_p}{\sigma_p} = \frac{0.8 \cdot 10^9 \times 3.86 \cdot 10^{-19}}{5 \cdot 10^{-16}} = 3.1 \mathrm{MW/cm}^2
\end{aligned} \tag{1}$$

Since the cross section is 30 $\mu\mathrm{m}^2$, the pump power should be 925 mW.

$$(1-T)e^{\alpha\ell} = 1 \tag{2}$$

which leads, with $\ell = 0.02cm$, to $\alpha = 11.157$ cm^{-1}. Therefore, $\Delta N_{eq} = 11.157/\sigma = 2.231 \cdot 10^{16}$ cm^{-3}. Since $\Delta N_0 = -6 \cdot 10^{17}$ cm^{-3}, $\Delta N_{eq}/\Delta N_0 = -0.03719$.

$$\begin{aligned}
\Delta N_{eq} &= \Delta N_0 + R' T_p = 2.231 \cdot 10^{16} \\
R' &= -R\Delta N_0 \\
(1 - RT_p) &= \frac{\Delta N_{eq}}{\Delta N_0} = -0.03719 \\
R &= \frac{1}{T_p}(1 - \frac{\Delta N_{eq}}{\Delta N_0}) = \frac{1.03719}{T_1} + \frac{1.03719R}{2} \\
0.481R &= \frac{1.0372}{T_1} \\
R &= 0.862 \cdot 10^9 \mathrm{s}^{-1} = \sigma_p I_p/(h\nu_p) \\
I_p &= \frac{Rh\nu_p}{\sigma_p} = \frac{0.862 \cdot 10^9 \times 3.86 \cdot 10^{-19}}{10^{-16}} = 3.33 \mathrm{MW/cm}^2
\end{aligned} \tag{3}$$

Since the cross section is 30 $\mu\mathrm{m}^2$, the pump power should be 998 mW.

Thus the pump intensity is $6/(30 \cdot 10^{-8}) = 20$ MW. The pump rate is:

$$R = \sigma_p I_p/(h\nu_p) = 5.18 \cdot 10^9 \mathrm{s}^{-1}. \tag{4}$$

We have to find the T_p, the new saturation intensity I'_s that this rate corresponds to.

$$\frac{1}{T_p} = \frac{1}{T_1} + \frac{R}{2} \tag{5}$$

which gives $T_p = 3.34 \cdot 10^{-10}$ s. The new saturation intensity is considerably higher:

$$I'_s = \frac{T_1}{T_p} I_s = 7.48 I_s \tag{6}$$

where I_s is the "natural" saturation intensity $= W_s/T_1 = h\nu/(2\sigma T_1) = 0.135$ MW/cm^2. since $R' = -R\Delta N_0 = 3.11 \cdot 10^{27}$, we have the "equilibrium" inversion that corresponds to the small signal gain (no laser intensity):

$$\Delta N_e = T_p R' + \Delta N_0 = 4.38 \cdot 10^{17}, \tag{7}$$

which should, through saturation, reduce to the same value than at threshold:

$$\begin{aligned}
\frac{\Delta N_e}{1+\frac{I}{I_s'}} &= 2.231 \cdot 10^{16} = \frac{4.38 \cdot 10^{17}}{1+\frac{I}{I_s'}} \\
\frac{I}{I_s'} &= \frac{43.8}{2.231} - 1 = 18.6. \\
IA &= 18. \times 7.48 \times 0.13510^6 \times 30 \cdot 10^{-8} = 5.45\text{W}
\end{aligned} \tag{8}$$

which gives an output power of $5.45 \times .2 = 1.1.$ W.