Homework 5 + solution
Due Wednesday, before 1 p.m., November 15, 2023

1 Fabry-Perot

1.1 Empty cavity

Find the linewidth, free spectral range of the transmission modes of the Fabry-Perot with the following parameters:

1. thickness 1 mm.
2. 2 mirrors with equal (intensity) reflectivity of R = 99%
3. $\lambda = 500$ nm
4. Index of refraction $n = 1$

Free spectral range: $\Delta\nu_{fsr} = c/2\ell = 1.5 \cdot 10^{11}$ HZ.
In radian/s: $9.42 \cdot 10^{11}$ S^{-1}
Finesse:

$$F = \frac{\pi\sqrt{R}}{1-R} = 313$$

which makes the FWHM of the transmission 0.48$\cdot 10^9$ Hz.
In radian/s: $3.015 \cdot 10^9$.

1.2 Dispersion due to an absorber

The Fabry-Perot cavity is filled with an absorbing medium with an homogeneously broadened absorption line exactly resonant with a mode of this Fabry-Perot.
Linear absorption coefficient $\alpha_0 = 1$ mm^{-1}
The inverse linewidth of the line (phase relaxation time) is $T_2 = 1$ ps.
Calculate the contribution of this line to the index of refraction $n(\Delta\omega)$ where $\Delta\omega$ is the detuning from the center of the line.

$\frac{d\mathcal{E}}{dz} = \mathcal{E}_0 e^{-\alpha z/2}$ For a Lorentzian line:

$$\alpha/2 = \frac{\alpha_0/2}{1+\Delta\omega^2 T_2^2}$$

which is the real part of the complex function

$$\frac{\alpha_0/2}{1+i\Delta\omega T_2}$$

which has as imaginary part

$$\frac{(\alpha_0/2)\Delta\omega T_2}{1+\Delta\omega^2 T_2^2}$$

which is the dispersion function to be identified with $k(\Delta\omega) = \frac{2\pi}{\lambda} n(\Delta\omega)$. We thus have:

$$n(\Delta\omega) = \frac{\lambda\alpha_0}{4\pi}\frac{\Delta\omega T_2}{1+\Delta\omega^2 T_2^2}.$$

1.3 Absorber in the Fabry-Perot

Calculate how the transmission of laser light for the mode at resonance and for two adjacent modes is affected by the absorber inserted in the cavity.

In the lecture on Fabry-Perot of October 16, slide 15, we derived the transmission function of a Fabry-Perot with gain:

$$|\mathcal{T}|^2 = \frac{(1-R)^2}{(1-Re^a)^2} \frac{1}{1 + \frac{4Re^a}{(1-Re^a)^2}\sin^2\frac{\delta}{2}}$$

We have here absorption: $a = -\alpha_0 \ell$.
The peak transmission is no longer 1, but

$$\frac{(1-R)^2}{(1-Re^a)^2} = \frac{(1-0.99)^2}{(1-0.99e^{-0.5})^2} = 0.000626$$

a very small number, because the *effective* path between the high reflectivity mirrors is very long, compared with the linear absorption length.

For the second part of this question, we neglect the frequency shift of the mode due to the dispersion of the absorption line. But the absorption is now $a = -\alpha(\Delta\omega)\ell$ with $\Delta\omega = 9.42 \cdot 10^{11}$ S^{-1} and $\Delta\omega T_2 = 9.42 \cdot 10^{11}$ $\mathrm{S}^{-1} T_2 = 3.77$. The value of a for the adjacent modes is $a = -\frac{\alpha_0 \ell}{1+14.2} = -0.0658$. The change in transmission for the two adjacent resonances is:

$$\frac{(1-R)^2}{(1-Re^a)^2} = \frac{(1-0.99)^2}{(1-0.99e^{-0.0658})^2} = 0.0187$$

1.4 Calculate the shift in frequency for the two modes adjacent to the center mode

The change in phase δ is $2\Delta k\ell = 2k\ell\Delta n = \frac{4\pi\ell}{\lambda}\Delta n$ where

$$\Delta n = \frac{\lambda\alpha_0}{4\pi}\frac{\Delta\omega T_2}{1+\Delta\omega^2 T_2^2}.$$

$$\delta = \ell\alpha_0 \frac{\Delta\omega T_2}{1+\Delta\omega^2 T_2^2} = \ell\alpha\Delta\omega T_2 = 0.24.$$

The shift in frequency is

$$\delta\Delta\omega/2\pi = \Delta\omega\alpha\Delta\omega T_2/2 = 9.442 \cdot 10^{11} \times 0.24/(2\pi) = 36\text{GHz}.$$