Laser Physics I (Physics 464)
Homework I, due Monday, August 31, 12 PM (noon), 2026

1 Second order propagation equation

Maxwell's propagation equation:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2} \tag{1}$$

1.1 From susceptibility to index of refraction

For a plane wave along z:

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2} \tag{2}$$

For linear polarization $P_L = \epsilon_0 \chi E$, show that this equation is equivalent to:

$$\frac{\partial^2 E}{\partial z^2} - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \tag{3}$$

with $n =?$

1.2 From second order propagation equation to first order

Same problem as above, except that you use the slowly varying approximation to derive:

$$\left(\frac{\partial \mathcal{E}}{\partial z} + \frac{n}{c}\frac{\partial \mathcal{E}}{\partial t}\right) = 0 \tag{4}$$

Do you find the same expression for n as in Section 1.1?

1.3 Find the linear susceptibility

Using the results of Sections 1.1 and 1.2 find the linear susceptibility χ for fused silica ($n = 1.5$) and zinc selenide ($n = 2.6$). Comment on the differences.

Solution

1.4 From susceptibility to index of refraction

Maxwell's propagation equation (plane wave along z):

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2} \tag{5}$$

For linear polarization $P_L = \epsilon_0 \chi E$

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = -\mu_0 \omega^2 \epsilon_0 \chi E. \tag{6}$$

which is equivalent to:

$$\frac{\partial^2 E}{\partial z^2} - \mu_0 \epsilon_0 (1+\chi) \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \tag{7}$$

or

$$\frac{\partial^2 E}{\partial z^2} - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \tag{8}$$

with

$$n = \sqrt{1+\chi} \tag{9}$$

k vector:

$$\frac{\omega}{c}\sqrt{1+\chi}$$