

Laser Physics I (Physics 464)
Homework II, due Monday, September 14, 2026

1 Shortest possible pulse

The shortest physically possible pulse should have its spectrum composed of positive frequencies. That excludes Gaussian and secant hyperbolic shaped pulses that extend from $-\infty$ to $+\infty$. A triangular pulse (Fig. 1) with its Fourier transform extending from 0 to $2\omega_0$ is theoretically possible. ω_0 is the central average (angular) frequency corresponding to the wavelength of 800 nm.

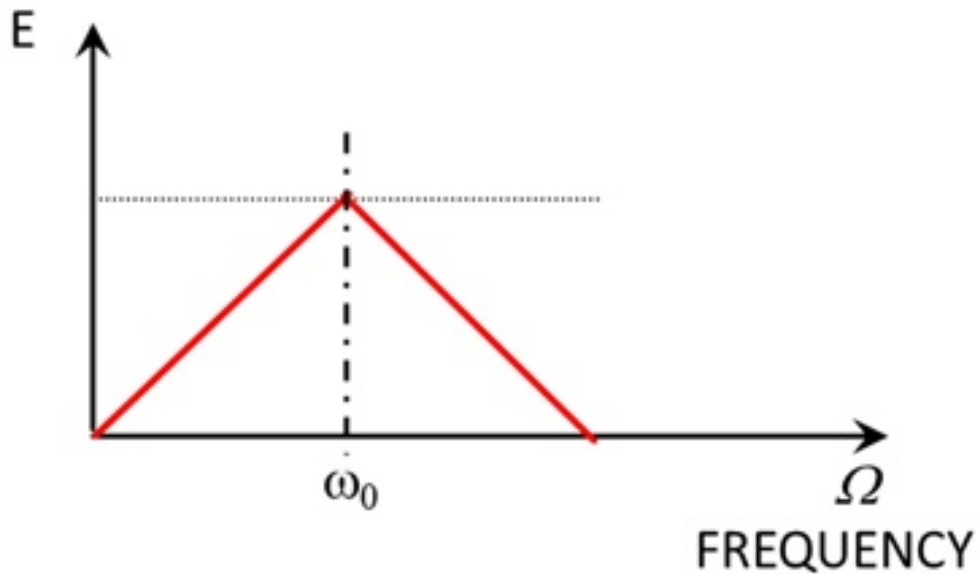


Figure 1: Spectral amplitude of an ultrashort pulse.

1. Find the temporal shape $E(t)$ of this pulse

Use properties of the Fourier transform.

Consider the triangle centered at zero frequency. That triangle is the autocorrelation of two square spectra of width ω_0 . The inverse Fourier trans-

form of one square spectrum is $E(t) \propto \text{sinc}(\frac{\omega_0 t}{2})$.

The inverse F.T. of the autocorrelation of the zero centered triangle is thus $E(t) \propto \text{sinc}^2(\frac{\omega_0 t}{2})$.

But the spectrum is not centered at zero. Apply the shift property to find the pulse shape:

$$E(t) \propto \text{sinc}^2(\frac{\omega_0 t}{2}) e^{i\omega_0 t}$$

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2. Decompose the field in amplitude and phase factor at the central frequency

It is propagating in a medium with index of refraction

$$n = n_0 + a\Omega$$

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- Can you define a group velocity? **Yes:**

$$\frac{1}{v_g} = \frac{dk}{d\Omega} = \frac{n}{c} + \frac{2a\omega_0}{c}.$$

- Using Maxwell's equation reduced to first order, how does this pulse propagate?

$$\tilde{E}(t, z) = \frac{\sin[\omega_0(t - \frac{z}{v_g})]}{\omega_0(t - \frac{z}{v_g})} e^{i(\omega_0 t - k_0 z)}$$

- Numerical example: $a = \frac{1}{20\omega_0}$

3. Propagate the pulse using the second order Maxwell propagation equation

Take the Fourier transform of Maxwell propagation equation.

Decompose in forward/backward.

Keep the forward:

$$\left(\frac{\partial}{\partial z} + i \frac{n(\Omega)\Omega}{c} \frac{\partial}{\partial t} \right) E = 0$$

Substitute n and integrate:

$$E(z, \Omega) = E(0, \Omega) e^{i(-\frac{2a}{c}\Omega z - a\Omega^2)}$$