Laser Physics I (Physics 464)
Homework III, due Monday, October 19, 2026

1 Evolution of the pulse energy

Find an evolution equation for the pulse energy
$W = \frac{1}{2}\epsilon_0 cn \int_{-\infty}^{\infty} \mathcal{E}^2 dt = \frac{1}{2}\sqrt{\epsilon/\mu_0} \int_{-\infty}^{\infty} \mathcal{E}^2 dt$.
integrate the field envelope propagation over time, after multiplying both sides by $\mathcal{E}$ and using the third Bloch equation.
Why does it make physical sense?

2 Find an evolution equation for the pulse average frequency

$\dot{\varphi}\rangle$ is the average phase derivative (average deviation of the frequency from ω_ℓ):

$$\langle\dot{\varphi}\rangle = \frac{\int_{-\infty}^{\infty} \mathcal{E}^2\dot{\varphi}dt}{\int_{-\infty}^{\infty} \mathcal{E}^2 dt} = \frac{1}{2W}\left[\sqrt{\frac{\epsilon}{\mu_0}}\int_{-\infty}^{\infty} \mathcal{E}^2\dot{\varphi}dt\right]$$

Multiply both sides by W and take the derivative

$$\begin{aligned}\frac{d(W\langle\dot{\varphi}\rangle)}{dz} &= W\frac{d\langle\dot{\varphi}\rangle}{dz} + \langle\dot{\varphi}\rangle\frac{dW}{dz} \\ &= \frac{1}{2}\sqrt{\frac{\epsilon}{\mu_0}}\frac{d}{dz}\int_{-\infty}^{\infty} \mathcal{E}^2\dot{\varphi}dt\end{aligned}$$

The last term can be calculated inserting the time derivative of the phase propagation equation in this equation.

More hints? Ask!