Laser Physics 464 — Homework 2

Due Wednesday, September 18, 2024

Fourier Transform review

1. Applying properties of linearity, shift to calculate Fourier transforms

4 pointsUsing the linearity property of the Fourier transform and the Fourier transform of the square function, find the Fourier transform of the function defined by:

$$\begin{aligned} E(t) &= 0 \qquad && t < 1 \\ &= 1 && 1 < t < 2 \\ &= 2 && 2 < t < 3 \\ &= 1 && 3 < t < 4 \\ &= 0 && t > 4 \end{aligned} \tag{1}$$

Solution Problem 1

We construct the Fourier transform starting from a square pulse of amplitude A, centered at zero, width 1, which is the sinc:

$$E_a(\Omega) = a\frac{\sin\frac{\Omega}{2}}{\frac{\Omega}{2}} \tag{2}$$

The three square pulses, if centered at zero, have as Fourier transform:

$$E_2\Omega + E_1(\Omega)[2\cos\Omega] \tag{3}$$

The ensemble being shifted by 2.5 time units:

$$E(\Omega) = 2\left\{(1+\cos\Omega)\frac{\sin\frac{\Omega}{2}}{\frac{\Omega}{2}}\right\}e^{2.5i\Omega}. \tag{4}$$

2. Fourier transform of a triangle

4 points Find the Fourier transform of an isosceles triangle. (Hint: think autocorrelation)

Solution Problem 2

sinc^2 since it is the autocorrelation of a rectangle.

3. Time-Bandwidth Product

A Gaussian (chirped) pulse has the field amplitude:

$$\tilde{\mathcal{E}} = \mathcal{E}_0 e^{-(1+ia)(\frac{t}{\tau_G})^2}. \tag{5}$$

Disregarding the amplitude factors, show that the Fourier transform of the chirped Gaussian is:

$$\tilde{\mathcal{E}}(\Omega) = e^{-\Omega^2 \tau_G^2/4(1+a^2)}. \tag{6}$$

Find the time bandwidth product for the Gaussian pulse if

(a) The width is defined by the FWHM of the intensity — find $\tau_{FWHM} \times \Omega_{FWHM}$ 4 points.

(b) The width is defined by the mean square deviation (MSQ).4 points Find $\langle \tau_{MSQ} \rangle \times \langle \Omega_{MSQ} \rangle$.

Derive the FWHM of the pulse intensity.

Solution Problem 3

If we re-write the Gaussian of Eq. (6) as:

$$\tilde{\mathcal{E}}(\Omega) = e^{-\Omega^2/W^2}, \tag{7}$$

we know from the previous derivation that:

$$\Omega_{FWHM} = W\sqrt{2\ln 2} = 2\frac{\sqrt{1+a^2}}{\tau_G}\sqrt{2\ln 2} \tag{8}$$

The time-bandwidth product in terms of the FWHM is thus:

$$4\ln 2\sqrt{1+a^2} \tag{9}$$

To define the width by MSQ, we note that $\langle t\rangle = \langle \Omega\rangle = 0$ because the pulses are symmetric and centered on the origin. We have thus only to calculate:

$$\begin{aligned}\langle t^2\rangle &= \frac{\int_{-\infty}^{\infty} t\tilde{\mathcal{E}}(t)t\tilde{\mathcal{E}}(t)^* dt}{\int_{-\infty}^{\infty} |\tilde{\mathcal{E}}(t)|^2 dt} = \frac{\left(\frac{\tau_G}{\sqrt{2}}\right)^3 \int_{-\infty}^{\infty} x^2 e^{-x^2} dx}{\left(\frac{\tau_G}{\sqrt{2}}\right) \int_{-\infty}^{\infty} e^{-x^2} dx} \\ &= \frac{\tau_G^2}{2}\frac{\sqrt{\pi}/2}{\sqrt{\pi}},\end{aligned} \tag{10}$$

and a similar expression in the frequency domain, except that τ_G is replaced by W given above. This leads to:

$$\langle t^2\rangle\langle\Omega^2\rangle = \frac{1}{4} \tag{11}$$

$$\tau_{FWHM} = \tau_G\sqrt{2\ln 2}. \tag{12}$$

The time for which half of the peak intensity is reached is given by:

$$-\ln 2 = -\frac{2t_{HW}^2}{\tau_G^2} \tag{13}$$

The FWHM is twice t_{HW}, or:

$$\tau_{FWHM} = \tau_G\sqrt{2\ln 2}. \tag{14}$$

4. The shortest possible pulse

4 points Central frequency corresponding to 800 nm; square spectrum from 0 to 2 ω_0. It propagates in a dielectric of which the index of refraction varies linearly from $n = 1$ at zero frerquency to $n = 1.5$ at 800 nm. Find (solve) the propagation equation for this pulse.

solution on ppt file.