Laser Physics 464 — Homework 2

Due Wednesday, September 18, 2024

Fourier Transform review

1. Applying properties of linearity, shift to calculate Fourier transforms

Using the linearity property of the Fourier transform and the Fourier transform of the square function, find the Fourier transform of the function defined by:

$$\begin{aligned} E(t) &= 0 && t < 1 \\ &= 1 && 1 < t < 2 \\ &= 2 && 2 < t < 3 \\ &= 1 && 3 < t < 4 \\ &= 0 && t > 4 \end{aligned} \tag{1}$$

2. Fourier transform of a triangle

Find the Fourier transform of an isosceles triangle. (Hint: think autocorrelation)

3. Time-Bandwidth Product

A Gaussian (chirped) pulse has the field amplitude:

$$\tilde{\mathcal{E}} = \mathcal{E}_0 e^{-(1+ia)(\frac{t}{\tau_G})^2}. \tag{2}$$

Disregarding the amplitude factors, show that the Fourier transform of the chirped Gaussian is:

$$\tilde{\mathcal{E}}(\Omega) = e^{-\Omega^2 \tau_G^2 / 4(1+a^2)}. \tag{3}$$

Find the time bandwidth product for the Gaussian pulse if

(a) The width is defined by the FWHM of the intensity — find $\tau_{FWHM} \times \Omega_{FWHM}$.

(b) The width is defined by the mean square deviation (MSQ). Find $\langle \tau_{MSQ} \rangle \times \langle \Omega_{MSQ} \rangle$.

Derive the FWHM of the pulse intensity.

4. The shortest possible pulse

Central frequency corresponding to 800 nm; square spectrum from 0 to 2 ω_0. It propagates in a dielectric of which the index of refraction varies linearly from $n = 1$ at zero frerquency to $n = 1.5$ at 800 nm. Find (solve) the propagation equation for this pulse.