# Laser Physics 464 — Homework 2

Due Monday, September 25, 2023

## Fourier Transform review

### 1. Fluorescence decay

An atomic system has been prepared in the upper state, and starts fluorescing at $t = 0$. We expect an exponential decay. The electric signal amplitude is $\mathcal{E}(t) = e^{-at}u(t)$ where "$a$" is positive real number and

$$\begin{aligned} u(t) &= 0 \qquad\qquad t < 0 \\ &= 1 \qquad\qquad t \geq 0. \end{aligned} \tag{1}$$

a) Find the Fourier transform of the signal, in the frequency domain. Do you expect it to be real?

b) Sketch the amplitude and phase of $E(\Omega)$

### 2. Applying properties of linearity, shift to calculate Fourier transforms

Using the linearity property of the Fourier transform and the fourier transform of the square function, find the Fourier transform of the function defined by:

$$\begin{aligned} E(t) &= 0 \qquad\qquad t < 1 \\ &= 1 \qquad\qquad 1 < t < 2 \\ &= 2 \qquad\qquad 2 < t < 3 \\ &= 1 \qquad\qquad 3 < t < 4 \\ &= 0 \qquad\qquad t > 4 \end{aligned} \tag{2}$$

## 3. Correlation and Convolution

Find the autocorrelation and autoconvolution of the function defined by:

$$\begin{aligned} E(t) &= 0 && t < 0 \\ &= 1 && 0 < t < 1 \\ &= 0.5 && 1 < t < 2 \\ &= 0 && t > 2 \end{aligned} \tag{3}$$

The time signal is a square pulse followed by a rectangle of height 1/2.

## 4. Fourier transform of a triangle

Find the Fourier transform of an isosceles triangle. (Hint: think autocorrelation)

## 5. Convolution

Polarization is not always an instantaneous response like assumed in $P(t) = \epsilon_0 \chi(t)\mathcal{E}(t)$ and the correct way to find the polarization is

$$P(t) = \epsilon_0 \int \chi(\tau)\mathcal{E}(t-\tau)d\tau \tag{4}$$

Where the electric field is a single pulse given as

$$\mathcal{E}(t) = e^{\frac{-t^2}{\tau_G^2}} \tag{5}$$

and the dielectric susceptibility is:

$$\begin{aligned} \chi &= 0 && t < 0 \\ &= e^{\frac{-t}{\tau}} \end{aligned} \tag{6}$$

What is the polarization as a function of frequency? Assuming the polarization corresponds to an absorption line, how does the real part of the polarization affect the transmitted pulse?

## 6. Time-Bandwidth Product

A Gaussian (chirped) pulse has the field amplitude:

$$\tilde{\mathcal{E}} = \mathcal{E}_0 e^{-(1+ia)(\frac{t}{\tau_G})^2}. \tag{7}$$

Disregarding the amplitude factors, show that the Fourier transform of the chirped Gaussian is:

$$\tilde{\mathcal{E}}(\Omega) = e^{-\Omega^2\tau_G^2/[4(1+a^2)]}. \tag{8}$$

Derive the FWHM of the pulse intensity.

Find the time bandwidth product for the Gaussian pulse if

(a) The width is defined by the FWHM of the intensity — find $\tau_{FWHM} \times \Omega_{FWHM}$.

(b) The width is defined by the mean square deviation (MSQ). Find $\langle\tau_{MSQ}\rangle \times \langle\Omega_{MSQ}\rangle$.

## 7. The poor man's attosecond source

A smart researcher created a train of ultrashort pulses by adding the to radiation of a Nd:YAG laser its second harmonic, third harmonic, fourth harmonic and fifth harmonic in phase. Assume all these harmonics have been reduced to the same amplitude. The Nd:YAG laser has a wavelength of 1.06 $\mu$ m. The n$^{th}4harmonicof4E$ has the field $E^n$.

1. What is the resulting field in time?

2. Assuming the 5 beams are made of pulses 2 ns wide, how does this affect the spectrum?