Laser Physics I — test I
Wednesday, 1 p.m., October 25, 2023

1 Isotope separation by radiation pressure

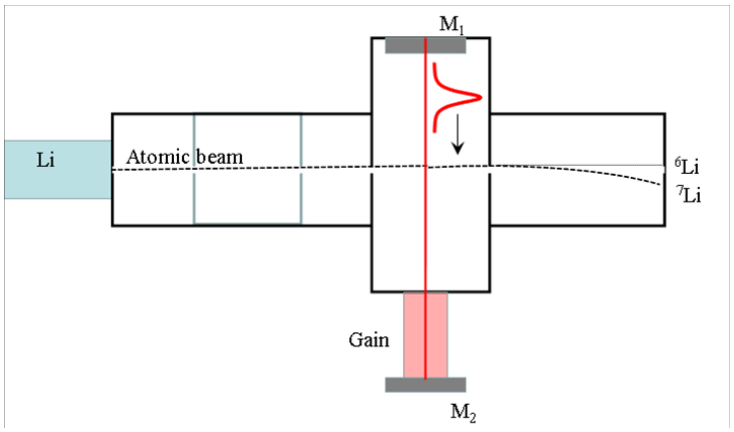

Figure 1: Deflection of atomic beam by radiation pressure.

Lithium has 2 isotopes, of atomic mass 6 and 7. Consider an atomic beam of lithium, irradiated transversely by pulses of frequency exactly resonant with the transition frequency of ^{7}Li around 670 nm. The light pulses travel back and forth in a laser cavity (mode-locked laser), in a time much shorter than the fluorescence lifetime T_1. In a steady state situation, one pulse (a "π" pulse) puts N atoms in the excited state, loosing N photons in the process. As it comes back after being reflected by mirror M_2, all N atoms are returned to the ground state, and the pulse has regained the N photons. No energy has been lost by the circulating pulse. The atoms have returned to the ground state, but they have gained kinetic energy through momentum exchange with the photon.
You have created energy!!!?????

Note that the number N is irrelevant in this problem, so you can choose $N = 1$ (interaction with a single atom).

1. Calculate the recoil velocity of one atom that has absorbed a photon of light.
2. Calculate the corresponding kinetic energy of that atom.
3. While investors may pay $$$$ for that scheme, explain how/why the energy is conserved, and what is the loss process.
4. Demonstrate quantitatively that the energy is conserved (at best).

1.1 Problem lithium - solution

The atomic mass of lithium is 7. The frequency corresponding to the wavelength of 670 nm is $\omega = 2\pi c/\lambda = 2.8 \cdot 10^{15}$ s^{-1}. The recoil velocity of the atom is calculated from the equality of momenta: $Mv = \hbar\omega/c$.
Lithium mass in kg:

$$M = \frac{7 \cdot 10^{-3}}{6.022 \cdot 10^{23}}$$

Photon energy: $1,24/0.67$ eV

$$v = \frac{\hbar\omega}{Mc} = \frac{1.0545 \cdot 10^{-34} \times 2.8 \cdot 10^{15} \times 6.022 \cdot 10^{23}}{7 \times 3 \cdot 10^{8} \times 10^{-3}} = 0.085\mathrm{m/s}. \tag{1}$$

The kinetic energy of the atom is:

$$M\frac{v^2}{2} = \frac{\hbar\omega}{Mc^2}\frac{\hbar\omega}{2} = 4.2 \cdot 10^{-29} J. \tag{2}$$

The fraction of the photon energy is $\frac{\hbar\omega}{Mc^2} = 3.06 \cdot 10^{-10}$.

The energy corresponding to the Doppler shift is:

$$\hbar\Delta\omega_D = \hbar\frac{v}{2c}\omega = \frac{\hbar\omega}{Mc^2}\hbar\omega = 4.2 \cdot 10^{-29}\mathrm{J}, \tag{3}$$

which is the single recoil energy. The factor 1/2 because we take the average between the velocity before and after the photon impact.

If the light pulse were to be sent back, the beam would recover a photon of a lower energy [by the amount given by Eq. (3)], while the atom would double its kinetic energy, and the energy would be conserved. After a number of cavity round-trips, the atoms my no longer be at exact resonance with the laser radiation because of velocity of the lithium atoms.

Two level systems 9/13 slides 50-51; Review slide 9-10-13-14.

2 Phases

Consider the optical arrangement of Fig. 2(a) where a beam of field amplitude $\mathcal{E}_0$ is incident from the left. The beam splitter is 50%.

1. What is the (complex) field reflection $\tilde{\mathcal{R}}_0$ and transmission $\tilde{\mathcal{T}}_0$?
2. A thin saturable absorber with small signal absorption coefficient α_0 is inserted in the ring [Fig. 2(b)]. What is the saturated absorption coefficient?
3. A beam is sent at normal incidence through an interface between two media of index n_1 and n_2 [Fig. 2(c)]. Find relations between the field reflection $\tilde{r}$ and transmission $\tilde{t}$.

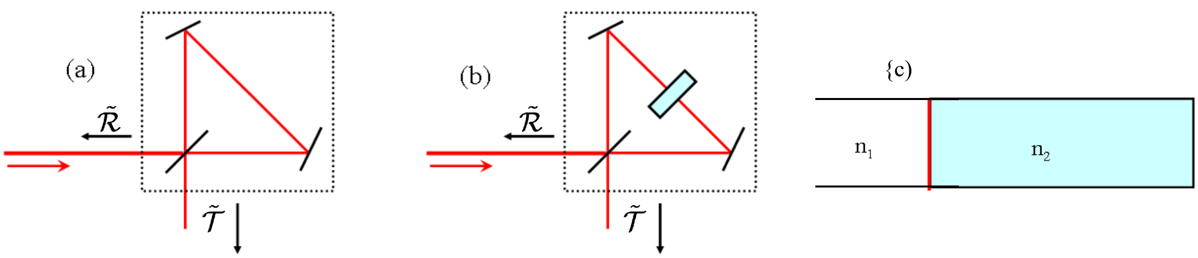

Figure 2: (a) What is the (complex) field reflection $\tilde{\mathcal{R}}_0$ and transmission $\tilde{\mathcal{T}}_0$? (b) Write an expression for the saturated absorption coefficient as a function if the intensity I incident from the left. (c) Find relations between the field reflection $\tilde{t}$ and transmission $\tilde{t}$ for a beam sent at normal incidence through the interface between media of index n_1 and n_2.

2.1 Solution

The Fresnel formulae apply to simple interface between two dielectrics. You cannot make a 50% beam splitter that way! The phase relations that we derived are more general and important. No interferometer would work without them.

(a)

This is nearly identical of the situations of slide 8 of the review. In slide 22 of the introduction (also slide 25) we derived from energy conservation:

$$\tilde{r}^*\tilde{t} + \tilde{r}\tilde{t}^* = 0$$

which implies:

$$\cos(\varphi_r - \varphi_t) = 0$$

meaning the phase shift in reflection and transmission are at 90 degrees from each other.

1. $\tilde{\mathcal{R}} = \tilde{r}\tilde{t} + \tilde{t}\tilde{r}$
   $\tilde{\mathcal{R}} = \tilde{r}^2 + \tilde{t}^2 = 0$ because these two terms are 180 degree out of phase (2×90 degrees). All the energy is thus reflected; none is transmitted.

2. Counter propagating beams:

$$\alpha = \frac{\alpha_0}{1 + 3\frac{I_i}{I_s}}$$

where $I_i = I/2$ is the intensity inside the ring. Slide 22 of the review.

3. Same derivations as in slide 8 of the review and slide 22 of the introduction, except that medium 2 has index n_2.

$$(r_{12}t_{21}^* + r_{12}^*t_{21}) + n_2(r_{21}t_{12}^* + r_{21}^*t_{12}) = 0$$

2.2 Fabry-Perot

Some of you picked out of a book the formula for intensity transmission. I insisted that this is not general, since the absolute value squared of the reflectivity does not carry any phase information!
Review slides 3 and 25. The Fourier transform of the single side exponential was treated in 2 homeworks.

The Fourier transform of $\mathcal{E}$ is:

$$\tilde{\mathcal{E}}(\Omega) = \int_0^\infty e^{-(0.1+i\Omega)t}dt = \frac{10}{1 + 10i\Omega} \tag{4}$$

where Ω is in units of 10^{12} s^{-1}. For the Fabry-Perot:

$$\mathcal{T} = \frac{(1-R)e^{id\Omega/c}}{1 - Re^{-2id\Omega/c}}. \tag{5}$$

We know that $d = 1$ mm, and the finesse is 100, which corresponds to R = 97%. The speed of light being 0.3 mm/ps, $d/c = 3.3333ps$. The Fabry-Perot transmission function is thus:

$$\begin{aligned} \mathcal{T} &= \frac{(1-R)e^{i3.33\Omega}}{1 - Re^{-i6.66\Omega}} \\ &\approx \frac{e^{i3.33\Omega}}{1 + 6.5i\Omega} \end{aligned}$$

where we have expanded the exponential to first order, and divided numerator and denominator by $(1 - R)$. The Fourier Transform of the transmitted field is:

$$\begin{aligned} \tilde{\mathcal{E}}_{tr} &= \frac{10}{1+10i\Omega}\frac{e^{i3.33\Omega}}{1+6.5i\Omega} \\ &= \frac{10e^{i3.33\Omega}}{1 + 16.5i\Omega - 65\Omega^2} \end{aligned} \tag{6}$$

The half-width of the two Lorentzians are $\Delta\Omega_p = 0.1$ ps^{-1} for the pulse, and $\Delta\Omega_{FP} = 0.15$ ps^{-1} for the Fabry-Perot. The product of both functions therefore does not extend beyond $\Delta\Omega = 0.03$ ps^{-1}. For these limits, we have:

$$65 \times (0.03)^2 \approx 0.06$$
$$6.5 \times 0.03 \approx 0.195$$

We can neglect the term 0.06 as compared to 1 + 0.195. With this approximation, the inverse Fourier transform of Eq. (6) is:

$$\mathcal{E}_{tr} = e^{-\frac{(t-3.33)}{16.5}}. \tag{7}$$

Note that the delay of 3.33 ps is simply the traversal time of the Fabry-Perot (d/c). The initial 10 ps pulse got stretched to 16.5 ps by the Fabry-Perot.